An Optimum Design of Error Diffusion Filters Using the Blue Noise in All Graylevels

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SUMMARY The error diffusion filter in this paper is optimized with respect to the ideal blue noise pattern corresponding to a single tone level. The filter coefficients are optimized by the minimization of the squared error norm between the Fourier power spectra of the resulting halftone and the blue noise pattern. During the process of optimization, the binary pattern power spectrum matching algorithm is applied with the aid of a new blue noise model. The number of the optimum filters is equal to that of different tones. The visual fidelity of the bilevel halftones generated by the error diffusion filters is evaluated in terms of a weighted signal-to-noise ratio, Fourier power spectra, and others. Experimental results have demonstrated that the proposed filter set generates satisfactory bilevel halftones of comparable visual quality of bilevel halftones and better power spectral characteristics than those in Ref. [8]. The tone-adaptive control of the filter coefficients is simple and fast; it is enough to read out an entry in a linear coefficient array of which entry is indexed by the grayscale expressed in digital count.

1. Introduction

Digital halftoning methods [1]–[18] can make a significant difference in printed or displayed image quality. Halftone images resemble the original continuous-tone images, when viewed from a distance due to the low-pass filtering nature of the human visual system (HVS) [13]–[16], [19]–[22]. Digital halftoning is used to render a continuous-tone image on printers, computer monitors or other displaying devices that are capable of producing a few levels of tone. The perceptual quality of the halftoned image depends on the spatial distribution of the bilevel dots. The error diffusion algorithms [1]–[9] among various halftoning techniques [10]–[18] have been proposed for realizing the homogeneous dot distribution and sharp rendering of bilevel images.

It is known that error diffusion halftoning produces unwanted textures such as worm artifacts at highlight and shadow regions and banding effects at mid-tone regions. Banding effects can strongly appear at so-called key levels [4], [8]. They are basically defined by multiples of the reciprocals of small integers, if the continuous-tone resides in the interval of [0, 1].

The error diffusion halftoning presented by Ostro-
2. Error Diffusion Halftoning

2.1 A Description of the Error Diffusion Algorithm

The continuous-tone input and the bilevel output are assumed as \( x(m, n) \in [0, 1] \) and \( y(m, n) \in \{0, 1\} \), where \( m \) and \( n \) stand for the horizontal and vertical locations, respectively. The error diffusion halftoning shown in Fig. 1 is expressed by the following equation system.

\[
\begin{align*}
  a(m, n) &= x(m, n) + \sum_{k, \ell \in \Omega} w_{k, \ell}^{(i)} d(m - k, n - \ell) \\
  y(m, n) &= \lfloor a(m, n) + 0.5 \rfloor \\
  d(m, n) &= a(m, n) - y(m, n)
\end{align*}
\]

where \( \lfloor \cdot \rfloor \) represents the maximum integer that does not exceed the argument. \( \Omega \) stands for the neighborhood where the quantization error is diffused. \( w_{k, \ell}^{(i)} \) denotes a coefficient of the error diffusion filter for the \( i \)th level in the tone intensity. The error diffusion filter has three coefficients in this work. The error is diffused at the east, southwest, and south locations with respect to the location of the target pixel being quantized at present. Pixels are visited in serpentine scanning to reduce the chance of worm patterning.

The behavior of error diffusion can be analyzed in the frequency domain. To do that, the quantization is represented by a linear stochastic model shown in Fig. 2, where \( e(m, n) \in (-0.5, 0.5) \) is a random variable with an independent \(^1\) uniform distribution, and represents the quantization error. Applying the quantization model to Eq. (2) and applying the two-dimensional \( z \)-transform to Eqs. (1)–(3), one obtains

\[
\begin{align*}
  A(z_1, z_2) &= X(z_1, z_2) + W^{(i)}(z_1, z_2)D(z_1, z_2) \\
  Y(z_1, z_2) &= A(z_1, z_2) + E(z_1, z_2) \\
  D(z_1, z_2) &= A(z_1, z_2) - Y(z_1, z_2).
\end{align*}
\]

The augmented input and the diffusion signal, \( A \) and \( D \), are eliminated to obtain

\[
Y = X + (1 - W^{(i)}) E. \tag{7}
\]

It is found that the low frequency components of the quantization error are swept out to high frequencies \([2]\), as far as the error diffusion filter is a lowpass filter and is of unity-gain at \( z_1 = z_2 = 1 \). This implies that

\[
w_{1,0}^{(i)} + w_{-1,1}^{(i)} + w_{0,1}^{(i)} = 1. \tag{8}
\]

Since \( 1 - W^{(i)} \) is a two-dimensional highpass filter, error diffusion halftones possess a blue noise-like characteristic by the second term in the right-hand side of Eq. (7). Also, \( 1 - W^{(i)} \) takes different gains depending on the spatial directions. As a result, banding artifacts at key levels can be induced at mid-tone areas, if the area is uniform and large enough to trigger locking to regular and stable patterning \([4]\). In the highlight and shadow areas, since the probability of the minority pixels is very low, the direction-dependent delivery of the quantization error can be visible as the worm textures, where a few bilevel dots align with a particular orientation. The worm patterning effect is weak in the case of serpentine scanning, because the error diffusion directions are alternated along the horizontal direction line by line.

2.2 Characterization of Blue Noise

A uniform bilevel image of which graylevel is \( g \in [0, 1] \) on average is considered. Pixels are assumed to be located on the rectangular sampling grids of which spacing is \( S^{\dagger\dagger\dagger} \). In the ideal blue noise pattern, the average sample distance between two minority pixels is characterized by the principal wavelength \([10],[11]\) defined by

\[
\lambda(g) = S / \sqrt{\min(g, 1 - g)} \tag{9}
\]

where \( \min(\cdots) \) denotes the minimum in the entries. Note that the minority pixels are distributed as uniform as possible, while the probability with respect to a minority pixel such that no more minority pixels are found within a distance of \( \lambda(g) \) is high unlike the white noise.

The blue noise is also characterized by the radially averaged power spectral density \(^{\dagger\dagger\dagger}\) (RAPSD) \([10]–[12],[14]\) as a function of the radial frequency, \( f_r \). The principal frequency \([10],[11]\) defined by

\[
X^2(f_r) = \frac{1}{N(\rho(f_r))} \sum_{k, \ell \in \Delta f_r} |X(k, \ell)|^2
\]

where \( X(k, \ell) \) is the DFT of \( x(m, n) \).
3. Design of the Error Diffusion Filters

3.1 Design of Blue Noise Patterns

Mitsa and Parker [27] presented the BIPPSM algorithm to make a bilevel blue noise pattern. Starting from a given bilevel white noise pattern, the bilevel pattern is updated to reach a blue noise pattern by a sequence of iterative procedures.

\[ f(g) = 1/\lambda(g) \]

plays an important role. The RAPSD of a blue noise pattern has a peak at \( f_r = f(g) \) as illustrated in Fig. 3. It shows a rapid attenuation as the radial frequency decreases from \( f(g) \), and is zero at \( f_r = 0 \). It approaches a fixed value of 1 in the frequencies above \( f(g) \). The unity-level at high frequencies owes to the normalization by \( g(1-g) \) [10], [11] that is the variance of the white noise pattern of which graylevel is \( g \).

The radially symmetric cut-off property in the two-dimensional power spectrum is also a consequence of the basic characteristics. Visually-pleasing bilevel patterns are desirable to acquire these features, which is one of the topics in the following section.

3.2 Optimum Design of Error Diffusion Filters

Every error diffusion filter is designed for a different grayscale level. The goal is to obtain a set of error diffusion filters every of which generates a bilevel halftone showing the blue noise characteristics, when a continuous-tone image of a specific grayscale level is halftoned by using the filter. Possible guidelines for obtaining satisfactory filters are summarized into the following three points.

\[ \int_0^{\infty} \frac{P^2(f_r)}{g(1-g)} df_r = \frac{1}{\sqrt{2}} \]  

The solution is given in Appendix A. While the principal frequency is scaled down by \( \sqrt{2} \) in Ref. [27], such an experimental remedy is unnecessary owing to the theoretical peaking built in Eq. (11).

Otherwise, the algorithm is terminated. In summary, the resulting bilevel pattern gains the spectral features of a desirable blue noise by its radial extension onto the 2-dimensional DFT domain. It also acquires the desirable minimum-distance limit property among minority pixels by swapping of pairs of zero and one.

Based on the characterizations of the blue noise in the previous section, a graphical user interface (GUI) has been developed to implement BIPPSM algorithm in MATLAB. The algorithm is however time-consuming and the peaking at \( f(g) \) is also missing in the RAPSD of an ideal blue noise in BIPPSMA [14], [27]. To overcome the weakness, the ideal RAPSD model for the blue noise is given by

\[ P(f_r) = \frac{\gamma_1}{\sqrt{g(1-g)}} \begin{cases} e^{f_r^2} - 1 & \text{for } f_r \leq f_g \\ (e^{-f_r^2} - 1) + 1 & \text{for } f_r \geq f_g \end{cases} \]

where

\[ \gamma_1 = \frac{h}{e^h - 1} \]

\[ \gamma_2 = \frac{h - 1}{e^{-h} - 1} \]

and \( f_g = f(g) \). \( h \) stands for the peak height and its value is derived from an assumption of power conservation between the blue noise model and the ideal white noise pattern:

\[ \int_0^{\infty} \frac{P^2(f_r)}{g(1-g)} df_r = \frac{1}{\sqrt{2}} \]  

\[ \gamma_1 = \frac{h}{e^h - 1} \]

\[ \gamma_2 = \frac{h - 1}{e^{-h} - 1} \]  

\[ \int_0^{\infty} \frac{P^2(f_r)}{g(1-g)} df_r = \frac{1}{\sqrt{2}} \]  

When an RAPSD matters, “high” and “low” frequencies are mentioned with reference to the principal frequency.

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radially symmetric, and shows a highpass characteristic of which cut-off frequency is slightly lower than \( f(g) \).

**C3:** With respect to any minority pixel, the probability such that no more minority pixels are found within a distance of \( \lambda(g) \) is high.

The guideline \( C1 \) is the most basic property of the blue noise, and it is responsible for the desirable 2-dimensional DFT profile by the radial extension. Hence the guideline \( C2 \) is effective and enough during the optimization, as far as \( C1 \) is reflected in the DFT profile. The guideline \( C3 \) is implicitly taken into account in the optimization through the blue noise reference pattern designed by the method in Sect. 3.1. It is worth to note that \( C3 \) is expectable from the intrinsic nature of error diffusion that divides the quantization error into different pieces to scatter them around different locations.

Since the error diffusion filters should be identical for dark levels and light levels in the sense of minority pixels, we impose the condition that

\[
W^{(i)} = W^{(255-i)}
\]

where

\[
W^{(i)} = \left( w^{(i)}_{1,0}, w^{(i)}_{-1,1}, w^{(i)}_{0,1} \right).
\]

\( W^{(i)} \) is a vector notation for the coefficients of the error diffusion filter. The superscript in parentheses denotes the grayscale in 8-bit digital count.

The filter coefficients are optimized by minimizing the error between the power spectra of the generated error diffusion-halftone and the blue noise pattern via simulated annealing [29], [30]. The minimization of the filter coefficients for the \( i \)th grayscale is expressed by

\[
W^{(i)} = \arg \min_{W^{(i)}} \| Y(W^{(i)}) - B^{(i)} \|
\]

where \( Y(W^{(i)}) \) is the DFT of the bilevel halftone produced by the error diffusion and \( B^{(i)} \) is that of the blue noise pattern of the \( i \)th level, which has been obtained in the previous subsection. \( \| \cdot \| \) stands for the \( L_2 \) norm.

The initial values of filter coefficients are set as \( W^{(i)} = \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right) \). A 1024×1024-pixel uniform patch is used in course of the optimizations for individual tones.

The coefficient values obtained by the proposed optimum design are presented in Appendix B. As an example, the frequency responses of Ostromoukhov’s filter and the filter designed for the grayscale \( g = 80/255 \) are shown in parts (a) and (b) in Fig. 4, respectively. Since the former is a result of coefficients interpolation between two key levels of \( 77/255 \) and \( 85/255 \), these frequency responses are a little bit different.

### 4. Experimental Results

The design of bilevel blue noise patterns is evaluated before comparing halftone images, because the bilevel patterns provide the critical reference for the optimization of error diffusion filters.

#### 4.1 Design Validation of Bilevel Blue Noise Patterns

The design of a bilevel blue noise pattern is checked by several objective measures. Figure 5(a) is a design example of such a bilevel pattern. Since its grayscale is \( g = 227/255 \), \( \lambda(g) \approx 3.0 \) and \( f(g) \approx 0.33 \). As seen in the bilevel pattern and a magnified view in part (b), there are fewer minority pixels within a distance shorter than \( \lambda(g) \).

The spatial statistics are also checked by means of pair correlation\(^{11} \) of which definition is given in the literature [14]. The pair correlation is the occurrence frequencies of the same sort pixels, with respect to every minority pixel, in the annuli with different radii. If the pair correlation value is small at a distance, any more minority pixels are expected to be absent around individual minority pixels at that distance. A desirable pair correlation in the blue noise patterns is hence close to zero below the principal wavelength. The fact is evident in the pair correlation in part (c), where its value rapidly vanishes as the radial distance decreases below \( \lambda(g) = 3.0 \). On the other hand, the pair correlation is constant above \( \lambda(g) \). It is an evidence of a homogeneous distribution with respect to distance between minority pixels.

In the Fourier power spectrum shown in part (d), higher intensities are plotted in lighter and the circle indicates the equi-distant frequencies at \( f_r = f(g) \). The radial symmetry is observed to be satisfactory. No abnormal intensities corresponding to unwanted spatially-periodic textures are found. Favorable properties in RAPSD are seen in part (e), where peaking around \( f_r = 0.33 \) is apparent.

Isotropy validation in the bilevel pattern is checked by anisotropy [10], [11] in the radial frequency domain and directional distribution [14] in the spatial domain. Anisotropy is defined by the relative variance of the DFT components within different annuli to the RAPSD. If the anisotropy of a

\(^{11}\)Pair correlation is dissimilar to the correlation between continuous-tone pixels such as auto-correlation. The pair correlation represents the degree of popularity of the pairs of the same sort pixels between “black” and “white.”
bilevel pattern is low and flat over the radial frequencies, the pattern is said to be isotropic. It is evident that the anisotropy in part (f) is at the levels around \(-6\) dB and indicates fine isotropy.

Directional distribution is a measure of near and far distributions of bilevel dots. It is defined by the occurrence frequencies in fan-shaped directional sections of a few annuli specified by different radii\(^1\). The near and far directional distributions in different directions are shown in parts (g) and (h), where the radial length from the origin represents the value of directional distribution. The near and far ranges are meant by the distances shorter than \(2\lambda (g)\) in between minority pixels and those between \(2\lambda (g)\) and \(4\lambda (g)\), respectively. It is observed that the near directional distributions have the values around 0.8 to 1.1 and far ones are perfect, showing that the pattern is tolerably isotropic.

Some RAPSDs of designed blue noise patterns are plotted in Fig. 6, where vertical dashed lines indicate the locations of individual principal frequencies. The asymptotic approach to the unity-gain at the highest radial frequency is observed except for the case of \(f(g) = \frac{1}{\sqrt{2}}\) which will show a peaking instead of the asymptotic convergence.

### 4.2 Comparisons of Halftone Images

The performance of the proposed error diffusion is evaluated in comparison to two representative error diffusion algorithms in Refs. [1], [8]. The evaluation measures include visual inspections, Fourier power spectrum, RAPSD, pair correlation, and a weighted signal-to-noise ratio (WSNR) [21], [22]. WSNR is defined by

\[
\text{WSNR} = 10 \log_{10} \frac{255^2}{\text{WMSE}}
\]

where

\[
\text{WMSE} = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} V(m, n) \times |X(m, n) - Y(m, n)|^2.
\]

\(^1\)Since each section is defined by two radii, \(r_1\) and \(r_2\), and an angle \(\theta\), with a fixed angular step, say \(\pi/8\), the directional distribution is written by \(D(r_1, r_2, \theta)\), where \(r_1 < r_2\). \(D(0, r, \theta)\) and \(D(r_1, r_2, \theta)\) are referred to as the near range and far range distribution, respectively. If the value of a directional distribution is larger or smaller than one, the population of minority pixels is too many or too few, respectively.
Fig. 7  Results of different error diffusion halftoning algorithms applied to a grayscale ramp. Three columns from left to right show the results by Floyd-Steinberg, Ostromoukhov, and the proposed methods, respectively. (a), (b), and (c) in the top row are the halftones. (d), (e), and (f) in the second row are Fourier power spectra. (g), (h), and (i) in the third row are RAPSDs. (j), (k), and (l) in the bottom row are pair correlations.
Fig. 8  Results of different error diffusion halftoning algorithms applied to *Portrait*. Three columns from left to right show the results by Floyd-Steinberg, Ostromoukhov, and the proposed methods, respectively. (a), (b), and (c) in the top row are the halftones. (d), (e), and (f) in the second row are Fourier power spectra. (g), (h), and (i) in the third row are RAPSDs. (j), (k), and (l) in the bottom row are pair correlations.
512×512 pixels. Obtained bilevel halftones, Fourier power spectra, RAPSDs, and pair correlations are shown in Fig. 7, where left-to-right three columns correspond to those of Floyd-Steinberg [1], Ostromoukhov [8], and the proposed method, respectively. The bilevel halftones are shown in parts (a), (b), and (c) on the top row, where WSNR values are given. Among them, part (c) shows less artificial patterning and is more visually-pleasing.

Parts (d), (e), and (f) show the 2-dimensional plots of the Fourier spectra of the resultant halftones, where stronger components are displayed in lighter. The horizontal line across the origin shows the image content corresponding to the horizontal gradation of the ramp. The other components are the quantization errors caused by halftoning. A directional bias is observed in part (d). Two vertical distributions around ±0.5 Hz in the horizontal frequency in part (e) have been considerably suppressed in part (f).

Clear differences are observed in RAPSD as found in parts (g), (h), and (i) on the third row in Fig. 7. Since the average principal frequency of the ramp image is calculated as $\bar{f}_g = \int_{0}\sqrt{g}dg + \int_{0.5}^{1} \sqrt{1-g}dg = 0.47$, the RAPSD in part (i) is found to have the desirable radial cut-off response around $f_r = \bar{f}_g$. On the other hand, the RAPSD peak in part (h) is located around $f_r = 0.59$, and (g) shows an excessive growth in high frequencies. It is evident that a comprehensive distribution of radial frequency components in part (i) is closer to the blue noise characteristics than that in part (g) and (h).

The pair correlation statistics are shown in parts (j), (k), and (l). In part (j), the peak is located around the radial distance of 1.8, which is shorter than $\bar{\lambda}_g = 1/\bar{f}_g = 2.1$. Both of the pair correlations in parts (k) and (l) are satisfactory.

The error diffusion bilevel halftoning has been applied to a natural image, Portrait, and the results are shown in Fig. 8. Parts (a), (b), and (c) are parts of the halftone images. The WSNR values are 32.63, 33.25, and 33.53 in dB in the cases of Floyd-Steinberg, Ostromoukhov, and the proposed, respectively. Banding artifacts are visually observed in the area at the top-left behind the head in part (a), while they are considerably suppressed in part (c). The power spectra shown in parts (d), (e), and (f) are similar to those in Fig. 7; a directional bias in part (d), vertical distributions in part (e), and improved radial symmetry in part (f). It implies that the high-frequency noise involved with the proposed halftoning is closer to the blue noise than those produced by the other two schemes.

Since RAPSD and pair correlation are statistical measures and since the target image in this experiment is a natural image with a variety of variations, precise comparisons on these statistics are more or less significant. In spite of the complex situation, a few remarks can be drawn regarding RAPSDs and pair correlations on the bottom two rows in Fig. 8. Note that the average principal frequency and average principal wavelength of the test image are calculated as $\bar{f}_g = 0.52$ and $\bar{\lambda}_g = 1.9$, respectively. As for RAPSD, no peaking at $\bar{f}_g$ is observed in part (g), and the peak location in part (h) is higher than $\bar{f}_g = 0.52$, while the peak in (i) is identical to $\bar{f}_g$. The pair correlations in parts (j) and (l) are fine showing a peak at 1.9 in radial distance, but the peak location in part (k) is farther than $\bar{\lambda}_g$. The overall behaviors of these measures obtained by the proposed method mimic the blue noise characteristics better than those by the other two methods.

Visual fidelity in terms of WSNR is experimented on 8-bit grayscale images of Airplane, Boat, Butterfly, Goldhill, Lena, Monarch, Peppers, Portrait, and Tiffany. The grayscale data is the luma defined by ITU-R BT.601 [31]. WSNR values are plotted in Fig. 9, where the proposed bilevel halftoning outperforms the other two methods.

As for 24-bit RGB color images, the proposed error diffusion halftoning has been separately applied to three color components. The WSNRs have been averaged over three cases of Floyd-Steinberg, Ostromoukhov, and the proposed.

5. Conclusions

We have presented an extension of the error diffusion halftoning in Ref. [8]. Bilevel blue noise patterns have been actually generated by BIPPSM algorithm combined with a new RAPSD model of blue noise. A set of error diffusion filters have been optimized for individual blue noise patterns.
with different tone intensities. The visual fidelity has been tested by means of several objective measures. The bilevel halftones produced by the proposed error diffusion halftoning are more favorable than those by two typical error diffusions that have been experimented in this work.

More in-depth investigations are necessary for color error diffusion halftoning for printing, because many aspects\(^1\) should be taken into account to reduce unwanted effects unique to color digital halftoning [13], [14], [16].

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References


\(^1\)They include dot arrangement between dot-on-dot or dot-off-dot, dot profile, dot gain, device profile, and tone/color reproductions. Most of them could be investigated if a target printing system would be well defined.

Appendix A: The Solution of Eq. (14)

Substituting Eq. (11) to Eq. (14) and carrying out the integration, one obtains a quadratic equation with respect to \( h \) as follows.

\[
a(h - 1)^2 + 2b(h - 1) + c = 0 \quad (A-1)
\]

where

\[
a = 1 - \frac{1}{d_1} + \frac{f_0}{d_1^2} - \frac{1}{d_2} + \frac{f_0}{d_2^2} \\
b = 3 - \frac{1}{d_1} + \frac{f_0}{d_1^2} - \frac{f_0}{d_2^2} \\
c = \frac{1}{2} - \frac{1}{d_1} + \frac{f_0}{d_1^2} - \frac{f_0}{d_2^2} \\
d_1 = c^{h_0} - 1 \\
d_2 = c^{h_1} - 1
\]
Table A.1  Coefficients of the Error Diffusion Filters. The index \( i \) stands for the graylevel in 8-bit digital count. Note that \( w_{1,1}^{i} = 1 - w_{1,0}^{i} - w_{-1,1}^{i} \).

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Fig. A 1  Coefficient values of the error diffusion filters. The shaded and unshaded stripes indicate the intervals specified by a part of extended key levels.

Appendix B: The Optimum Filters

The coefficients of the error diffusion filters optimized for individual 8-bit tones are listed in Table A.1. They are also plotted in Fig. A.1 for ease of viewing the variations in the filter coefficients.

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\[
f_h = \frac{1}{\sqrt{2}} - f_g.
\]

Since \( h \geq 1 \), the solution to Eq. (A·1) is obtained as

\[
h = 1 + \frac{-b + \sqrt{b^2 - 4ac}}{a}.
\]
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