Analysis and Relative Evaluation of Connectivity of a Mobile Multi-Hop Network

Keisuke NAKANO†‡, Member, Kazuyuki MIYAKITA†, Student Member, Masakazu SENGOKU†, and Shoji SHINODA††, Fellows

SUMMARY In mobile multi-hop networks, a source node S and a destination node D sometimes encounter a situation where there is no multi-hop path between them when a message M, destined for D, arrives at S. In this situation, we cannot send M from S to D immediately; however, we can deliver M to D after waiting some time with the help of two capabilities of mobility. One of the capabilities is to construct a connected multi-hop path by changing the topology of the network during the waiting time (Capability 1), and the other is to move M closer to D during the waiting time (Capability 2). In this paper, we consider three methods to deliver M from S to D by using these capabilities in different ways. Method 1 uses Capability 1 and sends M from S to D after waiting until a connected multi-hop path appears between S and D. Method 2 uses Capability 2 and delivers M to D by allowing a mobile node to carry M from S to D. Method 3 is a combination of Methods 1 and 2 and minimizes the waiting time. We evaluate and compare these three methods in terms of the mean waiting time, from the time when M arrives at S to the time when D starts receiving M, as a new approach to connectivity evaluation. We consider a one-dimensional mobile multi-hop network consisting of mobile nodes flowing in opposite directions along a street. First, we derive some approximate equations and propose an estimation method to compute the mean waiting time of Method 1. Second, we theoretically analyze the mean waiting time of Method 2, and compute a lower bound of that of Method 3. By comparing the three methods under the same assumptions using results of the analyses and some simulation results, we show relations between the mean waiting times of these methods and show how Capabilities 1 and 2 differently affect the mean waiting time.

key words: mobile multi-hop network, connectivity analysis, mobility, epidemic routing

1. Introduction

A mobile multi-hop network is constructed by two basic functions of mobile devices, namely direct communication and relaying. Here, direct communication is communication between two nodes through a wireless link between them, and relaying means to receive data from an adjacent node and forward the data to another adjacent node immediately. These two functions make it possible to construct a wireless multi-hop path between a source node S and a destination node D. As a result, S can transmit data to D after constructing a multi-hop path by using a certain routing protocol [1], [2]. Let M be a message destined for D. When M arrives at S, we sometimes encounter a situation where S and D have no multi-hop path between them because of the randomness of the positions of mobile nodes and their finite communication ranges. In this case, S cannot send M to D immediately; however, S can send M to D after waiting until a multi-hop path appears between S and D owing to a capability of mobility to reconstruct a connected multi-hop path by changing topology of the network during the waiting time. In this paper, we call this type of data transmission Method 1, and call the above capability of mobility Capability 1.

Even if there is no connected multi-hop path between S and D when M arrives at S, it is also possible to deliver M from S to D by epidemic routing [3]. Epidemic routing delivers M to D by making mobile nodes move M closer to D, and can be achieved in various manners. For example, if S as well as mobile nodes repeatedly distribute a copy of M to other mobile nodes in their communication ranges by direct communication, it is expected that some of the copies will approach D as the mobile nodes move and finally arrive at D in the near future. As can be seen from this example, epidemic routing does not require a connected multi-hop path between S and D to deliver M from S to D. If a node that directly received M from S can enter the communication range of D after moving, this node can move M closer to D by itself and can deliver M to D by direct communication. In this paper, Method 2 is epidemic routing that sends M from S directly to a mobile node and makes this mobile node send M directly to D after moving as in this example. We call the capability of mobility to move M closer to D Capability 2. Then, Method 2 does not use Capability 1 but Capability 2. Likewise, Method 1 uses Capability 1 rather than Capability 2.

Let \( T_w \) be the length of a duration from the time when M arrives at S to the time when D starts receiving M. Denote by \( E(\cdot) \) the expected value of \( \cdot \). If we use Methods 1 or 2 to deliver M from S to D, it is natural to compare the \( E(T_w) \) of Methods 1 and 2 to determine which is faster. At the same time, this comparison means a comparison between Capabilities 1 and 2 because Method 1 uses only Capability 1 and Method 2 uses only Capability 2 as mentioned. Furthermore, we can consider a combination of Methods 1 and 2 to reduce \( T_w \). Define Method 3 as a method that combines Methods 1 and 2 to minimize \( T_w \). Then, it is also interesting to investigate how much Method 3 reduces \( E(T_w) \) compared with Methods 1 and 2. For the above comparisons, analyses of the \( E(T_w) \)s of the three methods are needed.
Connectivity analysis is an important research issue in wireless multi-hop networks because the network is not always connected as mentioned, and the subject has been studied widely as surveyed in [4]. One of the purposes of connectivity analysis is to know in what situations the network is sufficiently connected. For example, this is achieved by finding the critical density of nodes (or the critical communication range) to achieve sufficient connectivity. Other than the density of nodes and communication range, the mobility of relay nodes is an important factor that affects connectivity in mobile multi-hop networks. The $E(T_w)$ of Method 1 is considered as a connectivity measure that reflects the effects of mobility because it means how long $S$ and $D$ have to wait until a connected multi-hop path appears between them due to the conditions of mobility. However, the $E(T_w)$ of Method 1 has never been analyzed theoretically even in a one-dimensional multi-hop network. In addition to this fact, comparing the $E(T_w)$ of Method 1 with those of Methods 2 and 3 has never been done. Therefore, analyses and relative evaluations of the $E(T_w)$s of the three methods are considered as a new approach to connectivity evaluation and are important from the viewpoint of connectivity analysis.

Also, analysis of $E(T_w)$ is important from the viewpoint of applications for the following reason. For applications that require low latency such as ordinary applications used in fixed computer networks, the probability that there is a multi-hop path between $S$ and $D$, denoted by $P_c$, is the most appropriate measure of connectivity, and maximizing $P_c$ is the most important task. However, long $T_w$ and low $P_c$ are sometimes acceptable in other kinds of applications that do not strictly require low latency, such as data collection through a sensor network, where $S$ and $D$ correspond to a sensor node and a data center, respectively. In such applications, $E(T_w)$ can be an appropriate measure of connectivity, and it is important to understand the behavior of $E(T_w)$ to guarantee the required connectivity performance.

With this as background, in this paper we analyze the $E(T_w)$s of Methods 1, 2 and 3 supposing that $S$ and $D$ are fixed nodes on a one-dimensional street and that there are two flows of mobile nodes on the street similar to pedestrian flows on a street. First, we approximately analyze the $E(T_w)$ of Method 1 in the one-dimensional network with the pedestrian mobility model because it is not easy to theoretically and exactly analyze the distribution, the mean and other characteristic values of the $T_w$ of Method 1 even in such a simple network. Similar issues of connectivity analyses using the pedestrian mobility model can be seen in [5]–[7]; however, these articles do not include analysis of the $E(T_w)$ of Method 1. Second, we theoretically analyze the $E(T_w)$ of Method 2, and propose a mathematical model to roughly compute a lower bound of the $E(T_w)$ of Method 3. By comparing the $E(T_w)$s of the three methods using the numerical results of the theoretical analyses and some simulation results, we show relations between the mean waiting times of the three methods and show how Capabilities 1 and 2 differently affect the mean waiting time.

In Sect. 2, we give definitions and assumptions. In Sect. 3, we explain background information and the problem statement. In Sect. 4, we analyze the mean waiting time of Method 1. In Sect. 5, we analyze the mean waiting times of Methods 2 and 3. We compare the mean waiting time of Method 1 with those of Methods 2 and 3 in Sect. 6. In Sect. 7, we conclude this paper.

2. Definitions and Assumptions

We consider transmission of a message $M$ from a fixed node $S$ to another fixed node $D$ on a one-dimensional street. The following are definitions and assumptions:

- Suppose that $S$, $D$ and mobile nodes are on a line. Let $x(n, t)$ be the coordinate of a node $n$ at time $t$ on the line. Let $L$ be the distance from $S$ to $D$. Suppose that $x(S, t) = 0$ and $x(D, t) = L$ for all $t$.
- Assume that a mobile node is a pedestrian moving along the line at a constant velocity $v$ without changing direction independently of other nodes as shown in Fig. 1. As represented in this figure, mobile nodes moving left (in the direction from $D$ to $S$) are represented by black nodes and those moving right (in the direction from $S$ to $D$) are represented by white nodes.
- Let $N_b$ and $N_w$ be the sets of mobile nodes moving left and those moving right, respectively. Denote a mobile node of $N_b$ by $b_i$, where $i$ is an integer such that $x(b_i, t) < x(b_{i+1}, t)$ for all $i$ and all $t$. Denote a mobile node of $N_w$ by $w_j$, where $j$ is an integer such that $x(w_j, t) > x(w_{j+1}, t)$ for all $j$ and all $t$. Note that $b_i$ enters the interval between $S$ and $D$ at $t = \frac{2L-b_i-vt}{v}$ and leaves this interval at $t = \frac{2L-b_i-w_{i-1}L}{v}$ if $x(w_i, t) < L$ and that $w_j$ enters the interval between $S$ and $D$ at $t = \frac{L-w_j-x_{i-1}}{-v}$ and leaves this interval at $t = \frac{L-w_j-x_{i+1}}{-v}$ if $x(w_j, t) > L$.
- Assume that the distributions of nodes of $N_b$ and $N_w$ obey Poisson distributions of intensity $\frac{1}{v}$ at the initial moment $t = 0$, respectively. Note that the same distributions are observed at time $t > 0$ because mobile nodes move independently from other nodes [8].
- Assume that there exists a wireless link between two nodes if the distance between them is not longer than $r$, where $r$ is a positive constant and $r < L$. Otherwise, there is no link between them.
- Let $G_b$ be the set of $(b_i, b_{i+1})$ such that $x(b_{i+1}, t) - x(b_i, t) > r$. Let $G_w$ be the set of $(w_j, w_{j+1})$ such that $x(w_{j+1}, t) - x(w_j, t) > r$.

Suppose that $M$ arrives at $S$ at $t = t_1$ and $S$ tries to send $M$ to $D$ immediately. For this purpose, we consider three methods denoted by Methods 1, 2 and 3 as mentioned in Sect. 1. The following defines the three methods in detail.

![Fig. 1](image-url) Fixed nodes S and D, and mobile nodes.
When we represent \( P_c \) and \( \tau \) and ends at \( \tau_2 \) appears. Define assume that an OFF period ends if a non-zero ON period the length of an ON period, which is a time period from the beginning of the OFF state to the end of the OFF state. Denote by \( \tau \) the length of an ON period, which is a time period from the beginning of the ON state to the end of the ON state. We assume that an OFF period ends if a non-zero ON period appears. Define \( T_{on} \) as the length of a time duration from \( \tau_1 \) to the time when \( D \) starts receiving \( M \) by Method 1 given that \( S \) and \( D \) are in the OFF state at \( \tau_1 \). Then, \( T_{on} \) is the time duration from \( \tau_1 \) to the time when the OFF state ends. Let \( P_c \) be the probability that there is a path between \( S \) and \( D \). When we represent \( P_c \) and \( T_{on} \) explicitly with parameters \( L, \lambda \) and \( \tau \), we represent them as \( P_c(L, \lambda, \tau) \) and \( T_{on}(L, \lambda, \tau) \), respectively.

Figure 2 shows an example of Method 1. In this example, \( M \) arrives at \( S \) at \( t = \tau_a \) during an OFF period that begins at \( \tau_a \) due to failure of the link between \( b_i \) and \( w_{j+1} \) and ends at \( \tau_d \). Then, \( T_{off} = \tau_d - \tau_a \) and \( T_{on} = \tau_d - \tau_c \).

**Method 2:** This method delivers \( M \) from \( S \) to \( D \) by allowing mobile nodes of \( N_w \) to carry \( M \) from \( S \) to \( D \). Suppose that \( x(w_j, \tau_1) \geq r \) and \( x(w_{j+1}, \tau_1) < r \). At \( \tau_1 \), immediately sends \( M \) directly to \( w_{j+1} \) if \( x(w_{j+1}, \tau_1) \geq r \), and \( S \) sends \( M \) to \( w_{j+1} \) when \( w_{j+1} \) enters the communication range of \( S \) otherwise. After receiving \( M \), \( w_{j+1} \) moves toward \( D \) and sends \( M \) directly to \( D \) when \( w_{j+1} \) enters the communication range of \( D \).

This method uses Capability 2 of mobility, which moves \( M \) closer to \( D \), and does not use Capability 1 as mentioned in Sect. 1. Note that it is sometimes possible for a mobile node to deliver the message to \( D \) while \( S \) and \( D \) are in an OFF period. In such a case, Method 2 outperforms Method 1.

**Method 3:** This method maximally utilizes both multi-hop transmission and epidemic routing. At \( \tau_1 \), \( S \) floods \( M \). Mobile nodes that receive \( M \) at \( \tau_1 \) or \( \tau_2 \) flood \( M \) repeatedly until \( M \) is delivered to \( D \). Suppose that a node can send \( M \) to every node in a connected component without delay in a flooding. Then, this method minimizes the time to deliver \( M \) to \( D \) although it may require a lot of redundant transmissions.

Define \( T_{w,2} \) as the length of a time duration from \( \tau_1 \) to the time when \( D \) starts receiving \( M \) by Method 2, and \( T_{w,3} \) as the length of a time duration from \( \tau_1 \) to the time when \( D \) starts receiving \( M \) by Method 3. Figure 3 is an example of Methods 2 and 3, where \( M \) arrives at \( S \) at \( t = \tau_e \). In this example, for Method 2, \( w_{j+1} \) carries \( M \) to \( D \), and \( T_{w,2} = \tau_f - \tau_e \). For Method 3, \( T_{w,3} \) receives \( M \) from \( S \) by a flooding at \( \tau_c \), and it sends \( M \) to \( D \) by another flooding at \( \tau_f \). Then, \( T_{w,3} = \tau_f - \tau_e \).

### 3. Problem Statement

In this paper, we analyze \( E(T_{w,1}) \) first. If the lengths of OFF periods are independent and identically distributed random variables, the OFF periods comprise a renewal process and we can use the well-known result on the mean residual lifetime in a renewal process [9] as follows:

\[
E(T_{w,1}) = \frac{E(T_{off}^2)}{2E(T_{off})}.
\]  

Although we cannot exactly say that they are independent in our model, we assume independence to simplify the analysis, and use Eq. (1) to approximately compute \( E(T_{w,1}) \). Before analyzing \( E(T_{off}) \) and \( E(T_{off}^2) \) to compute \( E(T_{w,1}) \), we explain the reason the analysis is difficult even in the one-dimensional network with the pedestrian mobility model and briefly explain our approach.

Consider a pair of nodes \( n_1 \) and \( n_2 \). If there is no node between \( n_1 \) and \( n_2 \) at time \( t \) and \( |x(n_1, t) - x(n_2, t)| > r \), we say that the interval between \( n_1 \) and \( n_2 \) is a gap at time \( t \). Here, \( S \), \( D \) and mobile nodes can be \( n_1 \) and \( n_2 \). Then, \( S \) and \( D \) are in the OFF state if and only if there is a gap between \( S \) and \( D \). This property can be seen easily.
Consider an example of an OFF period shown in Fig. 4. This OFF period begins right after $\tau_m$ and ends at $\tau_o$, where $\tau_m < \tau_o$. Figures 4(a) and (d) represent the networks at $\tau_m$ and $\tau_o$, respectively. Figures 4(b) and (e) show positions of black nodes at $\tau_m$ and $\tau_o$, respectively, and Figs. 4(c) and (f) show positions of white nodes at $\tau_m$ and $\tau_o$, respectively.

In this example, $(b_1, b_2) \notin G_b$, $(b_1, b_3) \in G_b$, $(b_2, b_3) \notin G_b$ and $(b_1, b_2) \notin G_b$ as represented in Fig. 4(b), and $(w_j, w_{j+1}) \notin G_w$, $(w_j, w_{j+1}+1) \notin G_w$, $(w_j, w_{j+1}+2) \notin G_w$ and $(w_j, w_{j+1}+3) \notin G_w$ as represented in Fig. 4(c). Denote $(b_1, b_2)$ and $(w_j, w_{j+1})$ by $g_1$ and $g_2$, respectively.

In Fig. 4(a), although there is a connected path between $S$ and $D$, the interval between $b_1$ and $w_1$ is being a gap because the length of this interval is equal to $r$ and will increase as $b_1$ and $w_1$ move. Then, an OFF period begins right after $\tau_m$. In this example, $S$ and $D$ are connected again and the OFF period ends when the distance between $b_1$ and $w_{j+1}$ becomes equal to $r$ as represented in Fig. 4(d). The OFF period continues while the length of the overlap of $g_1$ and $g_2$ is greater than $r$, and it ends when the length of the overlap becomes equal to $r$ as can be seen from Figs. 4(d), (e) and (f). Hence, in this example, we can simply represent the length of the OFF period by the relation between $g_1$ and $g_2$.

Consider another example of an OFF period depicted in Fig. 5. Figures 5(a) and (d) represent the networks at $\tau_p$ and $\tau_o$, respectively, where $\tau_o < \tau_p$. This OFF period begins right after $\tau_p$ and still continues at $\tau_p$. Figures 5(b) and (e) show positions of black nodes at $\tau_p$ and $\tau_o$, respectively, and Figs. 5(c) and (f) show positions of white nodes at $\tau_p$ and $\tau_o$, respectively.

In this example, $(b_1, b_2) \notin G_b$, $(b_1, b_2) \in G_b$, $(b_2, b_3) \notin G_b$ and $(b_1, b_2) \notin G_b$ as represented in Fig. 5(b), and $(w_{j-1}, w_j) \notin G_w$, $(w_{j-1}, w_j+1) \in G_w$, $(w_{j-1}, w_j+2) \in G_w$ and $(w_{j-1}, w_j+3) \in G_w$ as represented in Fig. 5(c). Denote $(b_1, b_2)$ and $(w_{j-1}, w_j+1)$ by $g_1$ and $g_2$, respectively.

As shown in the above examples, it is not easy to simply represent the condition in which an OFF period ends because we have to consider all possible combinations of $(b_1, b_2)$ and $(w_{j-1}, w_j+1)$ that cause successive appearances of gaps. Therefore, it is not easy to theoretically compute $E(T_{off})$ and $E(T_{off}^2)$ even in a one-dimensional network with the pedestrian mobility model.

To overcome this difficulty, we consider two special cases where $L < 2r$ (Case 1) and $L = 2r$ (Case 2). As will be explained later, the successive overlaps do not occur in Case 1. In Case 2, we have simple conditions that are satisfied at the end of an OFF period although the successive overlaps occur. Then, we analyze $E(T_{off})$ and $E(T_{off}^2)$ in Cases 1 and 2 theoretically although we use some approximations. For $L > 2r$ (Case 3), it is difficult to directly compute $E(T_{off})$ and $E(T_{off}^2)$. Then, we try to roughly estimate $E(T_{w1})$ using a property observed in the relation between $P_e$ and $E(T_{w1})$.
can be seen in [5]–[7]. In [5], the mean lengths of two types of ON periods and a lower bound of the mean length of an OFF period in multi-hop cellular networks were computed. In [7], \( T_{m} \) was analyzed in some cases; however, \( T_{w1} \) was not analyzed.

In this paper, there is only a requirement for the length of an ON period, namely \( T_{m} \) must be longer than 0 to terminate an OFF period. Although it is our future problem to analyze the waiting time until the appearance of an ON period that is longer than a desired value, it is expected that \( E(T_{w1}) \) computed in this paper is a lower bound of the mean of this waiting time.

Next, we theoretically analyze \( E(T_{w2}) \) under the same assumptions made for Method 1. Also, we propose a simple mathematical model of Method 3 to compute a lower bound of \( E(T_{w3}) \) because the theoretical analysis of \( E(T_{w3}) \) is difficult because Method 3 is a generalized version of Method 1. This mathematical model is derived from the results of the analyses of \( E(T_{w1}) \). From the numerical results of the analyses and some simulation results, we compare \( E(T_{w2}) \) and \( E(T_{w3}) \) with \( (1 - P_{c})E(T_{w1}) \) to compare the two capabilities of mobility. Note that \( E(T_{w1}) \) is the conditional mean of the waiting time given that S and D are in the OFF state at \( \tau_{1} \); therefore, we multiply \( (1 - P_{c}) \) to \( E(T_{w1}) \) for unconditioning and use \( (1 - P_{c})E(T_{w1}) \) for the above comparison. The mean waiting time of an epidemic routing in a one-dimensional network where mobile nodes move according to a Brownian motion model has been analyzed in [10]. However, we use the pedestrian mobility model because we analyze \( E(T_{w2}) \) and \( E(T_{w3}) \) to compare them with \( (1 - P_{c})E(T_{w1}) \) under the same assumptions, and it is more natural to assume the pedestrian mobility model as a mobility model on a street than the Brownian motion model.

In this paper, we observe in what situations \( T_{w1} \) rapidly increases, in what situations Method 2 outperforms Method 1, and how much Method 3 outperforms Method 1. For this purpose, we concentrate on situations where \( J \) is small and \( E(T_{w1}), E(T_{w2}) \) and \( E(T_{w3}) \) are large. Then, we neglect other short delays caused in a queue of packets or in retransmissions of packets as assumed in Sect. 2.

In this paper, it is also assumed that S can notice the beginning of every ON period in Method 1 and that S and D can notice every node entering their communication ranges in Method 2. Thus, these methods are assumed to be idealized ones in this paper, and it is necessary for S and D to use some additional procedures in actual situations; however, \( T_{w1} \) and \( T_{w2} \) are the minimum waiting times even in actual situations.

4. Mean Waiting Time of Method 1

4.1 Case 1

Suppose that an OFF period begins right after \( \tau_{a} \). In Case 1, there must be a pair of \( (b_{i}, b_{i+1}) \) \( \in G_{b} \) and \( (w_{j}, w_{j+1}) \) \( \in G_{w} \) that satisfies one of the following conditions at \( \tau_{a} \):

\[ A_{2}: \frac{L - r}{x(b_{i}, \tau_{a})} < L - r, \quad \frac{L - r}{x(w_{j}, \tau_{a})} < L - r \]

\[ A_{3}: x(w_{j}, \tau_{a}) = r, \quad x(b_{i}, \tau_{a}) = r \]

\[ A_{4}: 0 < x(b_{i}, \tau_{a}) < \frac{L}{x(w_{j}, \tau_{a})} < \frac{L}{x(w_{j}, \tau_{a})} \quad \text{and} \quad x(w_{j}, \tau_{a}) - x(b_{i}, \tau_{a}) = r \]

Note that these are mutually exclusive and cover all situations where an OFF period begins. In Case 1, an OFF period ends when a node enters the region \( [L - r, r] \) because a node has direct links to both S and D simultaneously while it is in \( [L - r, r] \). It also ends when a three-hop path is constructed even if there is no node in \( [L - r, r] \). We can easily see that we need at most three hops to connect S and D in Case 1. Then, one of the following events has to occur when the OFF period ends:

\[ B_{1}: b_{i+1} \text{ arrives at } x = r \]

\[ B_{2}: w_{j+1} \text{ arrives at } x = L - r \]

\[ B_{3}: \text{The distance from } b_{i+1} \text{ to } w_{j+1} \text{ becomes } r \]

Next, we theoretically analyze \( E(T_{w2}) \) and \( E(T_{w3}) \) under the same assumptions made for Method 1. Also, we propose a simple mathematical model of Method 3 to compute a lower bound of \( E(T_{w3}) \) because the theoretical analysis of \( E(T_{w3}) \) is difficult because Method 3 is a generalized version of Method 1. This mathematical model is derived from the results of the analyses of \( E(T_{w1}) \). From the numerical results of the analyses and some simulation results, we compare \( E(T_{w2}) \) and \( E(T_{w3}) \) with \( (1 - P_{c})E(T_{w1}) \) to compare the two capabilities of mobility. Note that \( E(T_{w1}) \) is the conditional mean of the waiting time given that S and D are in the OFF state at \( \tau_{1} \); therefore, we multiply \( (1 - P_{c}) \) to \( E(T_{w1}) \) for unconditioning and use \( (1 - P_{c})E(T_{w1}) \) for the above comparison. The mean waiting time of an epidemic routing in a one-dimensional network where mobile nodes move according to a Brownian motion model has been analyzed in [10]. However, we use the pedestrian mobility model because we analyze \( E(T_{w2}) \) and \( E(T_{w3}) \) to compare them with \( (1 - P_{c})E(T_{w1}) \) under the same assumptions, and it is more natural to assume the pedestrian mobility model as a mobility model on a street than the Brownian motion model.

In this paper, we observe in what situations \( T_{w1} \) rapidly increases, in what situations Method 2 outperforms Method 1, and how much Method 3 outperforms Method 1. For this purpose, we concentrate on situations where \( J \) is small and \( E(T_{w1}), E(T_{w2}) \) and \( E(T_{w3}) \) are large. Then, we neglect other short delays caused in a queue of packets or in retransmissions of packets as assumed in Sect. 2.

In this paper, it is also assumed that S can notice the beginning of every ON period in Method 1 and that S and D can notice every node entering their communication ranges in Method 2. Thus, these methods are assumed to be idealized ones in this paper, and it is necessary for S and D to use some additional procedures in actual situations; however, \( T_{w1} \) and \( T_{w2} \) are the minimum waiting times even in actual situations.
In this equation, \( \min(Y_1, Z_1) \) is an exponential random variable with the mean \( \frac{1}{\lambda} \). If \( Y_1 > Z_1 \), \( Y_1 - Z_1 \) is an exponential random variable with the mean \( \frac{1}{\lambda} \) because of the memoryless property. Then,

\[
E(T_{off} | A_1) = \frac{1}{v} \left( \frac{3}{2 \lambda} - \frac{1}{2 \lambda} e^{-\lambda(L-r)} \right)
\]

(5)

because the mean of \( \min \left( \frac{Y_1 - Z_1}{2}, L - r \right) \) given that \( Y_1 > Z_1 \) is equal to \( \frac{1}{2} \left[ 1 - e^{-\lambda(L-r)} \right] \) and \( P(Y_1 > Z_1) = \frac{1}{2} \). Also, we have

\[
E(T_{off}^2 | A_1) = \frac{E[\min(Y_1, Z_1)^2]}{v^2} + \frac{P(Y_1 > Z_1)}{v^2}
\]

\[
\times \left[ E \left( 2Z_1 \min \left( \frac{Y_1 - Z_1}{2}, L - r \right) \mid Y_1 > Z_1 \right) + E \left( \min \left( \frac{Y_1 - Z_1}{2}, L - r \right)^2 \mid Y_1 > Z_1 \right) \right]
\]

\[
= 4 - 2e^{-\lambda(L-r)} \left( L - r \right) e^{-\lambda(L-r)}
\]

(6)

because \( E[2Z_1 \min \left( \frac{Y_1 - Z_1}{2}, L - r \right) \mid Y_1 > Z_1] \) equals \( 2E[Z_1] \) if \( Y_1 > Z_1 \) because \( Z_1 \) and \( Y_1 - Z_1 \) are independent if \( Y_1 > Z_1 \), and the second moment of \( \min \left( \frac{Y_1 - Z_1}{2}, L - r \right) \) given that \( Y_1 > Z_1 \) equals \( \frac{2}{\lambda^2} - \frac{2(L-r)}{\lambda^2} e^{-\lambda(L-r)} - \frac{2(L-r)}{\lambda^2} e^{-\lambda(L-r)} \).

Next, suppose that an OFF period begins due to \( A_2 \). Define that \( Y_2 = x(b_i, \tau_a) - x(w_j, \tau_a) \) and that \( Z_2 = x(b_{i+1}, \tau_a) - x(w_j, \tau_a) \). Then, \( Y_2 \) and \( Z_2 \) are exponential random variables of intensity \( \frac{1}{\lambda} \). Let \( \alpha \) be \( x(b_i, \tau_a) - \frac{L}{2} \). Then,

\[
T_{off} = \min \left( Z_2 + \frac{L}{2} + \alpha, Y_2 + \frac{L}{2} + \alpha, Y_2 + Z_2 \right)
\]

\[
\approx \min \left( Z_2 + \frac{L}{2}, Y_2 + \frac{L}{2}, Y_2 + Z_2 \right)
\]

\[
= \frac{\min(Y_2, Z_2) + \min(Y_2 - Z_2, L/2)}{v}
\]

(7)

where we assume that \( \alpha \ll Y_2 \) and \( \alpha \ll Z_2 \) because \( 0 \leq \alpha < \frac{L}{2} \) and our main attention is paid to \( T_{off} \) for a small \( \lambda \) as mentioned. From Eq. (7), in the same manner as the derivations of \( E(T_{off} | A_1) \) and \( E(T_{off}^2 | A_1) \), we have

\[
E(T_{off} | A_2) \approx \frac{1}{v} \left( \frac{2}{\lambda} - \frac{1}{\lambda^2} \right)
\]

(8)

\[
E(T_{off}^2 | A_2) \approx \frac{6 - 4e^{-\frac{\lambda}{2L}}}{\lambda^2 v^2} - \frac{(L-r) e^{-\lambda(L-r)}}{\lambda^2 v^2}
\]

(9)

Consequently, we can approximately compute \( E(T_{w,1}) \) by Eqs. (1), (2), (3), (5), (6), (8) and (9).
a direct link between them and have direct links to S and D, respectively, after $b_k$ arrives at $x = r$ until $b_{k+1}$ arrives at $x = r$. Otherwise, $b_k$ and $b_{k+1}$ have no direct link between them; therefore, they cannot construct a three-hop path. Therefore, the above property holds.

Property 2: $w_t$ and $w_{t+1}$ construct a three-hop path with non-zero lifetime to connect S and D from $t = t_{k,t}$ to $t = t_{k,t} + \frac{(b_k, \ell, x) - x(b_k, \ell, x) - 2r}{r}$ if $x(w_t, t_a) - x(w_{t+1}, t_a) \leq r$, and they cannot construct such a three-hop path otherwise.

This can be proved in the same manner as Property 1. Also, we have:

Property 3: $b_k$ and $w_t$ construct a three-hop path with non-zero lifetime to connect S and D from $t = t_{k,t}$ to $t = t_{k,t} + \frac{(b_k, \ell, x) - x(b_k, \ell, x) - 2r}{r}$ if $x(w_t, t_a) - x(w_t, t_a) - 2r \leq r$, and they cannot construct such a three-hop path otherwise.

Proof. From the definition of $t_{k,t}$, we have $x(b_k, \ell, x) = \frac{(b_k, \ell, x) - x(b_k, \ell, x) - 2r}{r}$ and $x(w_t, t_a) = \frac{(b_k, \ell, x) - x(b_k, \ell, x) - 2r}{r}$. From these relations, if $|x(w_t, t_a) - x(b_k, t_a)| - 2r < r$, then $x(w_t, t_a) = x(b_k, t_a) + \frac{2r}{r}$. Also, we have $x(w_t, t_a) = x(w_t, t_a) + \frac{2r}{r}$. At $t = t_{k,t}$, therefore, $b_k$ connects $w_t$ and D, and $w_t$ connects $b_k$ and S. The lifetime of this three-hop path via $b_k$ and $w_t$ is not equal to 0 because it does not disappear at least until $b_k$ or $w_t$ arrives at $x = r$. If $|x(w_t, t_a) - x(b_k, t_a)| - 2r \geq r$, then $x(w_t, t_a) < x(b_k, t_a)$. From the definition of $t_{k,t}$, $b_k$ continues being linked directly to $b_k$ from $t = t_{k,t}$ to $t = t_{k,t} + \frac{2r}{r}$, however, S is connected to neither $b_k$ nor $w_t$ if $x(b_k, t_a) > 2r$, and D is connected to neither $b_k$ nor $w_t$ if $x(b_k, t_a) < r$. Therefore, $b_k$ and $w_t$ cannot construct any three-hop paths to connect S and D if $|x(w_t, t_a) - x(b_k, t_a)| - 2r \geq r$. Therefore, $b_k$ and $w_t$ construct a three-hop path to connect S and D at $t_{k,t}$ because $x(b_k, t_a) = r$ or $x(b_k, t_a) = 2r$; however, this path disappears right after $t_{k,t}$ and the lifetime of this path equals 0.

Let $s_1$ and $s_2$ be a sequence $x(b_{i+1}, t_a) - 2r, x(b_i, t_a) - 2r, \ldots$, and a sequence $-x(w_j, t_a), -x(w_{j+2}, t_a), \ldots$, respectively. We merge $s_1$ and $s_2$ into a single sorted sequence $s_1 = y_1, y_2, \ldots$ in ascending order. Let $n_m$ be the nth node that enters (0, L) after $t_a$, where $m = 1, 2, \ldots$. If $y_m$ corresponds to $x(b_k, t_a) - 2r$, where $k \geq i + 1$, $n_m$ is identical to $b_k$. If $y_m$ corresponds to $-x(w_t, t_a)$, where $\ell \geq j + 1$, $n_m$ is identical to $w_t$. Denote by $d_m$ the direction of $n_m$. Define that $U_0 = y_1$ and $U_m = y_{m+1} - y_m$ for $m = 1, 2, \ldots$. Then, $U_m$ is an exponential random variable with intensity $\lambda$ for all $m$.

We define the following events:

$C_m$: $U_m > r$; $C_m$: $U_m = r$; $d_m \neq d_m + 1$; $C_m$: $C_m \cup C_m$.

Then, from Properties 1 to 3, $n_m$ and $n_{m+1}$ construct a three-hop path that connects S and D after $t_a$, if $C_m$ occurs, and they cannot construct such a path if $C_m$ occurs. Define that $W = U_0$ if $C_1$ occurs, and that $W = \sum_{m=0} U_m$ if $(\cap_{m=1}^M C_m) \cap C_{M+1}$ occurs for $M \geq 1$. Here, $P(C_m) = P(U_m > r) = P(U_m \geq r) = e^{-\lambda r}$ for all $m$, and $E(U_m | U_m > r) = E(U_m | U_m \geq r) = r + \frac{1}{r}$. Therefore, $E(W) = E(U_0) P(U_1 \leq r)$

\begin{align}
&+ \sum_{M=1}^{\infty} \left( E(U_0) + ME(U_1 | U_1 > r) \right) P(U_1 > r)^M \\
&\times P(U_{M+1} \leq r) = r + \frac{1}{r} - e^{-\lambda r} - r.
\end{align}

(12)

We can compute $E(W^2)$ as follows because $U_0, U_1, \ldots, U_{M+1}$ are independent:

\begin{align}
E(W^2) &= E(U_0^2) P(U_1 \leq r) + \sum_{M=1}^{\infty} |E(U_0^2) \\
&+ ME(U_1^2 | U_1 > r) + 2ME(U_0)E(U_1 | U_1 > r) \\
&+ M(M-1)E(U_1 | U_1 > r)^2 \times P(U_1 > r)^M \\
&\times P(U_{M+1} \leq r) = \frac{2}{\lambda^2} + \frac{(r + \frac{1}{r})^2}{e^{\lambda r} - 1} + \frac{2(r + \frac{1}{r})^2}{e^{\lambda r} - 1}.
\end{align}

(13)

This formula is derived by using $\sum_{M=1}^{\infty} M(e^{-\lambda r}) = \frac{e^{-\lambda r}}{(1-e^{-\lambda r})^2}$ and $\sum_{M=1}^{\infty} M^2(e^{-\lambda r}) = \frac{e^{-\lambda r}(1+\lambda r)}{(1-e^{-\lambda r})^3}$.

Suppose that $(\cap_{M=1}^M C_m) \cap C_{M+1}$ occurs. Then, from Properties 1 to 3, any pair of nodes selected from $n_1, n_2, \ldots, n_{m+1}$ cannot construct any three-hop paths to connect S and D, and $n_{m+1}$ and $n_{m+2}$ can construct a three-hop path to connect S and D. If $d_{m+1} \neq d_{m+2}$, the OFF period ends when the distance between $n_{m+1}$ and $n_{m+2}$ becomes equal to $r$, and $T_{off} = \frac{W_{m+1}}{\lambda}$. If $d_{m+1} = d_{m+2}$ and $n_{m+1}$ and $n_{m+2}$ construct a three-hop path at $t = t_{a, \ell, w}$, however, $n_{m+1}$ may construct a three-hop path before $t = t_{a, \ell, w}$ together with $n_{m+2, \ell, \ell}$, where $\beta \geq 1$, if $d_{m+1} \neq d_{m+2}$ and $d_{m+2} = d_{m+1} \neq 1$. Then, $w_{m+1} < T_{off} \leq \frac{W_m}{\lambda}$ in this case. For a small $\lambda$, $W$ tends to be much larger than $r$ as can be seen from Eqs. (12) and (13), and our attention is paid to $T_{off}$ for a small $\lambda$. Hence, we approximately compute $T_{off}$ as $T_{off} \approx \frac{W_m}{\lambda}$ if $d_{m+1} = d_{m+2}$.

Then,

\begin{align}
E(T_{off} | A_m) &= \frac{E(W + \frac{W_m}{\lambda} | U_{M+1} \leq r)}{\lambda} \\
&\times P(d_{m+1} \neq d_{m+2}) + \frac{E(W + r)}{\lambda} \\
&\times P(d_{m+1} = d_{m+2}) = \frac{5e^{\lambda r} + 3\lambda e^{\lambda r} - 1}{4\lambda^2(e^{\lambda r} - 1)}.
\end{align}

(14)

where $E(W)$ is given in Eq. (12), $P(d_{m+1} \neq d_{m+2}) = \frac{1}{2}$, and $E(U_m | U_m < r) = \frac{1}{r} - \frac{1}{e^{\lambda r} - 1}$ for all $m$. Because $W$ and $U_{M+1}$ are independent, we can compute $E(T_{off} | A_m)$ with Eqs. (12) and (13) as follows:

\begin{align}
E(T_{off} | A_m) &= \frac{2 + e^{\lambda r}(7\lambda r(Ar + 2) - 8)}{8\lambda^2 r^2 (e^{\lambda r} - 1)^2}
\end{align}
+ \frac{e^{2\lambda r}[\lambda r(5\lambda r + 14) + 22]}{8\lambda^2 v^2(e^{\lambda r} - 1)^2}.

(15)

This equation is derived from \(E(W), E(W^2)\) and \(E(U_m^2|U_m < r) = \frac{2r}{\lambda} - \frac{2r^2}{\lambda(e^{r} - 1)}\). In Eqs. (14) and (15), \(W\) is a main factor that reflects the effects of successive overlaps of gaps, and such a factor does not appear in Case 1.

Next, we consider \(E(T_{off}|A_1)\) and \(E(T_{off}^2|A_1)\). In this case, an OFF period ends in various manners, and computations of \(E(T_{off}|A_1)\) and \(E(T_{off}^2|A_1)\) are more complicated than those of \(E(T_{off}|A_0)\) and \(E(T_{off}^2|A_0)\). To simplify the computation, we assume that \(x(b_t, \tau_a) = r/2\) and \(x(w_j, \tau_a) = 3r/2\) when \(A_1\) occurs, similar to the assumption made to compute \(E(T_{off}|A_2)\) and \(E(T_{off}^2|A_2)\). Owing to the above approximation, only \(b_{t+1}, b_{t+2}, \ldots\) and \(w_{j+1}, w_{j+2}, \ldots\) are candidates for relay nodes to connect \(S\) and \(D\) again. We redefine \(s_1, s_2, \text{ and } n_t\) as follows: Let \(s_1\) and \(s_2\) be a sequence \(x(b_{t+1}, \tau_a) - \frac{r}{2}, x(b_{t+2}, \tau_a) - \frac{r}{2}, \ldots\) and a sequence \(\frac{r}{2} - x(w_{j+1}, \tau_a), \frac{r}{2} - x(w_{j+2}, \tau_a), \ldots\), respectively. Let \(n_m\) be the \(m\)-th node that enters \((\frac{r}{2}, \frac{r}{2})\) after \(\tau_a\). For \(s_1, d_m, U_m\) and \(W\), we use the same definitions made for \(A_0\). Then, we can represent the condition to terminate an OFF period in a similar manner to \(A_0\), and the length of an OFF period is as follows:

\[
T_{off} \approx \begin{cases} 
\frac{w + \frac{U_{m+1}}{v}}{v}, & d_{M+1} \neq d_{M+2}, \\
\frac{w + 4}{v}, & d_{M+1} = d_{M+2}. 
\end{cases}
\]

(16)

Then,

\[
E(T_{off}|A_1) \approx E(T_{off}|A_0) - \frac{r}{2v} \\
= \frac{5e^{\lambda r} + \lambda r e^{\lambda r} + 2\lambda r - 1}{4\lambda t e^{\lambda r} - 1}.
\]

(17)

\[
E(T_{off}^2|A_1) \approx E(T_{off}^2|A_0) - \frac{r}{4} \{4E(W) + 2\lambda + E(U_{M+1}|U_{M+1} \leq r)\} \\
= \frac{2 + 2\lambda (\lambda r - 1) + e^{2\lambda r}[\lambda r(\lambda r + 4) + 22]}{4\lambda^2 v^2(e^{\lambda r} - 1)^2} \\
+ \frac{e^{\lambda r}[\lambda r(9\lambda r + 26) - 8]}{8\lambda^2 v^2(e^{\lambda r} - 1)^2}.
\]

(18)

Next, let us consider the expected values of \(T_{off}\) given that \(A_2\) occurs. In this case, \(b_t\) and \(w_{j+1}\) construct a three-hop path again when \(w_{j+1}\) arrives at \(x = r\). Also, \(w_{j+1}\) and \(b_{t+1}\) may construct it when the distance between them becomes equal to \(r\). Let \(Y_3\) and \(Z_3\) be \(r - x(w_{j+1}, \tau_a)\) and \(x(b_{t+1}, \tau_a) - 2r\), respectively. Then,

\[
T_{off} = \min \left(\frac{Y_3}{v}, \frac{Y_3 + Z_3}{2v}\right),
\]

(19)

where \(Y_3\) and \(Z_3\) are exponential random variables of intensity \(\frac{1}{v}\) and \(\frac{1}{2} < r\). Unlike \(A_0\) and \(A_2\), \(T_{off}\) does not include \(W\) in this case. Then, with some calculations, we have

\[
E(T_{off}|A_2) = \frac{4 - e^{-\lambda r} - 3e^{\lambda r} + \lambda r}{2\lambda t(1 - e^{\lambda r})}.
\]

(20)

In Case 2, we approximately compute \(E(T_{w1})\) by Eqs. (1), (10), (11), (14), (15), (17), (18), (20) and (21).

4.3 Numerical Results in Cases 1 and 2

The numerical results of the above equations in Cases 1 and 2 are shown in Figs. 6 and 7, respectively, together with computer simulation results to verify the equations. We compute \(E(T_{w1})\) for \(r = 10\) m, \(20\) m and \(50\) m with \(v = 1\) m/seconds, and three values of \(L\) for each \(r\) are evaluated in Case 1. It is intuitively expected that \(E(T_{w1})\) increases as \(\lambda\) decreases or \(r\) decreases. We use the mean number of nodes in the communication range \(r\), which is equal to \(\lambda r\), in the horizontal axis to observe effects of \(\lambda\) and \(r\) on \(E(T_{w1})\) simultaneously. In the computer simulations, we decide positions of mobile nodes randomly according to the same assumptions made in Sect. 2. If the system is in the OFF state at the initial moment, we observe the system at small intervals denoted by \(\Delta t\) and compute \(T_{w1}\) as the time length from the initial moment until the end of the OFF state. Otherwise, we initialize the positions of mobile nodes again. We repeat these procedures until we examine \(N\) different OFF periods, and compute the mean. In this paper,
Δt = 0.05 seconds and N = 10000. Because of iterations to observe the system state in each of the N OFF periods, the running time of a simulation is much longer than the time to compute the above equations.

From Fig. 6, we can confirm that the numerical results agree well with the computer simulation results in Case 1 although our analyses include some approximations. Also, from this figure, we can see that $E(T_{w,1})$ remains almost the same even if $L$ changes. From Fig. 7, we can confirm that, also in Case 2, the numerical results agree well with the computer simulation results and describe well the behavior of $T_{w,1}$ which rapidly increases as $\lambda$ decreases.

From Figs. 6 and 7, there is a big difference in $E(T_{w,1})$ between Case 1 and Case 2 even if the difference in $L$ is small. For example, we can observe such a result by comparing $E(T_{w,1})$ with $r = 10$ m and $L = 19.9$ m and that with $r = 10$ m and $L = 20.0$ m. It is considered that such a rapid increase of $E(T_{w,1})$ at the boundary between Case 1 and Case 2 is caused by $W$, which is a main factor in rapidly increasing $E(T_{w,1})$ in Case 2.

4.4 Case 3

As mentioned in Sect. 3, it is generally difficult to analyze $E(T_{w,1})$ in Case 3, where $L > 2r$. Hence, we try to characterize $E(T_{w,1})$ in Case 3 from a property observed in the simulation results. While we show $E(T_{w,1})$ in Case 1 with the horizontal axis of $\lambda r$ in Fig. 6, we show the same simulation results of $E(T_{w,1})$ with the horizontal axis of $P_r$ in Fig. 8. Also, we show simulation results in Cases 2 and 3 with the horizontal axis of $P_r$ in Fig. 9, where $r = 10$ m, 20 m and 50 m, and $L = 2r, 4r, 6r$ and 10r.

By using $P_r$ in the horizontal axis, we can find an interesting property as follows. In Fig. 8, we can see that $E(T_{w,1})$ decreases as $L$ increases in Case 1 while $P_r$ remains the same. In Cases 2 and 3, however, $E(T_{w,1})$ does not change so much even if $L$ increases, and is close to $E(T_{w,1})$ in Case 2 as shown in Fig. 9. Although it is intuitively expected that the tendency in Case 1 is different from those of Cases 2 and 3 because of the effects of the successive appearances of gaps, it remains as a future problem to find theoretical reasons the above property holds.

By assuming that the above property holds, it is expected that $E(T_{w,1}(L, \lambda, r)) \approx E(T_{w,1}(2r, \lambda_0, r))$ for $L > 2r$ if $P_r(L, \lambda, r) = P_r(2r, \lambda_0, r)$. We can compute $P_r(L, \lambda, r)$ by Eq. (4) because the distribution of all mobile nodes obeys a Poisson distribution of intensity $\lambda$ for $r \geq 0$ because mobile nodes move independently [8]. Then, we can estimate $E(T_{w,1}(L, \lambda, r))$ for $L > 2r$ as follows:

1. Compute $P_r(L, \lambda, r)$ by Eq. (4).
2. Solve $P_r(2r, \lambda_0, r) = P_r(L, \lambda, r)$ for $\lambda_0$.
3. Compute $E(T_{w,1}(2r, \lambda_0, r))$ by the equations derived in Sect. 4.2. This value is expected to be close to $E(T_{w,1}(L, \lambda, r))$.

Figure 10 shows the numerical results of $E(T_{w,1})$ computed by the above procedure together with simulation results, where $r = 10$ m. In this figure, we use $\lambda$ in the horizontal axis because only one value of $r$ is examined as opposed to Figs. 6 and 7. From the results, we can confirm that the numerical results agree well with the simulation results. Hence, using the above procedure, we can compute $E(T_{w,1})$ in Case 3 approximately but quickly compared with the computer simulation.
5. Mean Waiting Times of Methods 2 and 3

We suppose that a message for D arrives at S at time \( \tau_1 \). Suppose that \( x(w_{(j-1)}, \tau_1) \geq r \) and \( x(w_j, \tau_1) < r \). Let \( X_1 \) be the OFF period, and receive \( M \) when \( X_1 \). Let \( E \) denote this model by Method 3. Suppose again that the OFF period begins at \( \tau_1 \) to \( \tau_2 \). Let \( \lambda \) be a positive and small constant. We approximately compute \( E(T_{w,3}) \) by the equations to compute \( E(T_{w,1}) \) in Case 1 assuming that the message transmission between one pair of fixed nodes is independent from that between another pair of fixed nodes. Then,

\[
E(T_{w,3}) = K\{1 - P_c(L', \lambda, r)\}E(T_{w,1}(L', \lambda, r)) + \{1 - P_c(L - KL', \lambda, r)\} \times E(T_{w,1}(L - KL', \lambda, r)),
\]

where \( L' = 2r - \Delta x \) and \( K = \left[ \frac{1}{2r - \Delta x} \right] \).

6. Comparison between Methods 1, 2 and 3

In this section, we compare the mean waiting time of Method 1 with those of Methods 2 and 3. For Method 1, we use \( (1 - P_c)E(T_{w,1}) \) as the mean waiting time because \( E(T_{w,1}) \) is the mean waiting time given that \( S \) and \( D \) are not connected at \( \tau_1 \). We compute \( (1 - P_c)E(T_{w,1}) \) by the methods shown in Sect. 4 and by computer simulation. We compute \( E(T_{w,2}) \) by Eq. (22), \( E(T_{w,3}) \) by computer simulation, and \( E(T_{w,3a}) \) by Eq. (23).

Figure 11 shows \( (1 - P_c)E(T_{w,1}) \), \( E(T_{w,2}) \) and \( E(T_{w,3}) \) for \( L = 15 \) m and \( r = 10 \) m. All figures in this section use \( \lambda \) in the horizontal axis in the same manner as Fig. 10. This figure shows the relation between these three mean values for \( L < 2r \). In this figure, \( (1 - P_c)E(T_{w,1}) \) is the same as \( E_T(T_{w,3}) \) and is smaller than \( E(T_{w,2}) \). We can confirm that Method 1 can minimize the mean waiting time without any assistance from Capability 2 of mobility if \( L < 2r \).
Fig. 13 \((1 - P_c)E(T_{w,1}), E(T_{w,2}), E(T_{w,3})\) and \(E(T_{w,3a})\), where \(L = 40\) m and \(r = 10\) m.

Fig. 14 \((1 - P_c)E(T_{w,1}), E(T_{w,2}), E(T_{w,3})\) and \(E(T_{w,3a})\), where \(L = 100\) m and \(r = 10\) m.

Figures 12 to 14 show \((1 - P_c)E(T_{w,1}), E(T_{w,2})\) and \(E(T_{w,3})\) together with \(E(T_{w,3a})\), where \(L = 20\) m, \(L = 40\) m and \(L = 100\) m, respectively, and \(r = 10\) m. These figures show relations between the four mean values for \(L \geq 2r\). From these figures, we can see that \((1 - P_c)E(T_{w,1}), E(T_{w,2})\) and \(E(T_{w,3})\) increase as \(\lambda\) decreases and that \(E(T_{w,3})\) is smaller than \((1 - P_c)E(T_{w,1})\) and \(E(T_{w,2})\) for all \(\lambda\). For a large \(\lambda\), \((1 - P_c)E(T_{w,1})\) is almost the same as \(E(T_{w,3})\) and is smaller than \(E(T_{w,2})\); however, it rapidly increases as \(\lambda\) decreases and exceeds \(E(T_{w,2})\) at a certain \(\lambda\). As \(\lambda\) keeps decreasing, \(E(T_{w,2})\) approaches \(E(T_{w,3})\).

We describe the above relations with constants \(\Lambda_1\), \(\Lambda_2\) and \(\Lambda_3\). Suppose that \(E(T_{w,2})\) and \(E(T_{w,3})\) are considered as almost the same if \(\lambda \leq \Lambda_1\), that \([1 - P_c(L, \Lambda_2, r)]E(T_{w,3}(L, \Lambda_2, r)) = E(T_{w,2}(L, \Lambda_2, r))\) and that \((1 - P_c)E(T_{w,1})\) and \(E(T_{w,3})\) are considered as almost the same if \(\lambda \geq \Lambda_3\). Then, \(E(T_{w,3})\) is almost the smallest for \(\lambda \leq \Lambda_1\), \(E(T_{w,3}) < E(T_{w,2}) < (1 - P_c)E(T_{w,1})\) for \(\Lambda_1 < \lambda < \Lambda_2\), \(E(T_{w,3}) < (1 - P_c)E(T_{w,1}) < E(T_{w,2})\) for \(\Lambda_2 < \lambda < \Lambda_3\), and \((1 - P_c)E(T_{w,1})\) is almost the smallest for \(\lambda \geq \Lambda_3\). From these results, if \(\lambda \geq \Lambda_3\), Capability 1 works sufficiently well without any assistance from Capability 2.

The reverse is true if \(\lambda \leq \Lambda_1\). These results also show that we should utilize both Capability 1 and Capability 2 to reduce the waiting time if \(\Lambda_1 < \lambda < \Lambda_3\). Otherwise, Method 1 is inferior to a simple method like Method 2 if \(\lambda < \Lambda_2\).

From Figs. 12 to 14, we can confirm that the numerical results of Eq. (23) can be used as a lower bound of \(E(T_{w,3})\) in almost all cases although they become slightly greater than \(E(T_{w,3})\) in some cases because of the assumption that the multi-hop transmission between adjacent fixed nodes is independent from that between other adjacent fixed nodes. These figures also show that \(E(T_{w,3a})\) is close to \(E(T_{w,3})\) while \(\lambda \leq \Lambda_1\) or \(\lambda \geq \Lambda_3\). Then, from the numerical results of \((1 - P_c)E(T_{w,1}), E(T_{w,2})\) and \(E(T_{w,3a})\), we can estimate \(\Lambda_1\) and \(\Lambda_3\), which determine a range of \(\lambda\) in which both capabilities of mobility should be used, in addition to the fact that \(\Lambda_2\) can be computed by numerically solving an equation \((1 - P_c)E(T_{w,1}) = E(T_{w,2})\) for \(\lambda\).

7. Conclusions

In this paper, we considered the issue of connectivity of a mobile multi-hop network. As a measure of connectivity, we focused on the mean waiting time for delivery of a message from one fixed node to another, and considered the mean waiting times of three methods, Methods 1, 2 and 3. Method 1 uses Capability 1 of mobility, which reconstructs a connected multi-hop path by changing topology of the network during the waiting time. Method 2 uses Capability 2 of mobility, which moves a message closer to D during the waiting time. Method 3 maximally utilizes Capabilities 1 and 2 to minimize the waiting time.

We theoretically computed the mean waiting time of Method 1 in two cases and proposed a method to estimate it in another case. We theoretically computed the mean waiting time of Method 1 by comparing it with those of the other two methods using the results of the theoretical analyses and some computer simulation results. From this comparison, we showed that Method 1 minimizes the mean waiting time if \(L < 2r\). We also showed that there are four kinds of situations in which relations between Methods 1, 2 and 3 are different, and that effects of the two capabilities of mobility on reducing the mean waiting time are different between these situations. Using the numerical results of the above three theoretical analyses, we can estimate in what situations Method 1 and Method 2 can sufficiently reduce the mean waiting time, and in what situations a hybrid method such as Method 3 is necessary.

Other than the future problems described in previous sections, various extensions to other models such as two-dimensional network models and other mobility models are considered as future works.

Acknowledgments

This work was partially supported by the Ministry of Education, Science, Sports and Culture, Grant-in-Aid. The authors would like to thank anonymous reviewers for their valuable comments.
Consider a random variable $X$ that has the following density function given that $X > r$:

$$f_X(x) = \frac{\lambda}{e^{\lambda r} - 1} \left[ 1 + \sum_{j=1}^{[(x/r) - 1]} \frac{(-1)^j}{j!} (\lambda(x - (j + 1)r))^{j-1} \right] e^{-\lambda r} \{\lambda(x - (j + 1)r) + j\},$$

(A.1)

where $[\cdot]$ is the integer part of $\cdot$. This density function is that of the length of a clump defined in [11]. Then, for $L > r$,

$$P_c(L, \lambda, r) = 1 - P(X \leq L) = (1 - e^{-\lambda L}) \left( 1 - \int_r^L f_X(x)dx \right),$$

(A.2)

References