Simultaneous Measurement of Antenna Gain and Complex Permittivity of Liquid in Near-Field Region Using Weighted Regression

Nozomu ISHII, Member, Hiroki SHIGA, Naoto IKARASHI, Student Members, Ken-ichi SATO, Nonmember, Lira HAMADA, and Soichi WATANABE, Members

SUMMARY As a technique for calibrating electric-field probes used in standardized SAR (Specific Absorption Rate) assessment, we have studied the technique using the Friis transmission formula in the tissue-equivalent liquid. It is difficult to measure power transmission between two reference antennas in the far-field region due to large attenuation in the liquid. This means that the conventional Friis transmission formula cannot be applied to our measurement so that we developed an extension of this formula that is valid in the near-field region. In this paper, the method of weighted least squares is introduced to reduce the effect of the noise in the measurement system when the gain of the antenna operated in the liquid is determined by the curve-fitting technique. And we examine how to choose the fitting range to reduce the uncertainty of the estimated gain.

key words: SAR, near-field region, absolute gain, Friis transmission equation, least square method

1. Introduction

The mobile communication devices should be evaluated by their SAR (Specific Absorption Rate). In general, the value of SAR can be measured by an electric-field probe in the tissue-equivalent liquid [2]. The probe should be calibrated to relate received electric-field intensity to its output voltage. In 300 MHz–3 GHz, the calibration using the wave-guide system is currently recognized as standard according to the IEC document [2]. However, the mobile communication devices will be used over 3 GHz so that the effect of the attenuation should be included in the formula. As shown in Fig. 1, two identical antennas are assumed to be immersed and faced in the liquid with perfect polarization. The two antennas are connected to the ports of the vector network analyzer via semi-rigid cables. Looking enormous attenuation in the tissue-equivalent liquid so that the measurement cannot be realized in the far-field region of the antenna in the liquid. This means that the precondition of the Friis transmission formula is not suitable for our measurement. To overcome the difficulty, we proposed to extend this formula to be valid in the near-field region of the antenna [5], [6]. The merit of this extension is that it is only necessary to include one term or two terms of the asymptotic expansion in regression function and extra equipments, for example, high-power amplifiers, are not required to enhance the dynamic range of the measurement system. On the other hand, estimated gain and dielectric property depend upon the choice of the fitting range as well as the regression function. And, the behavior of one- or two-term asymptotic expansion is somewhat unstable for the regression process.

We have two techniques to determine the gain of the antenna in the liquid. One is to determine the gain after the dielectric property of the liquid is determined by the pre-experiment, for example, the dielectric probe method [6], [7]; the other is to determine the gain and the dielectric property simultaneously [4], [5]. The former is somewhat sensitive to the values of the measured complex permittivity of the liquid so that the near-field region gain does not always converge with the far-field gain. On the other hand, the latter can estimate the values of the gain and complex permittivity if the curve-fitting works well. In this paper, we introduce the method of weighted least square for the latter technique. The conventional method of least squares using no weight does not work below the noise level of the measurement system. The regression using the weight can lessen the effect of the noise in the measurement system. In addition, we can choose wide range of fitting so that the uncertainty of the measurement can be reduced.


To evaluate the value of the absolute gain of the antenna in the liquid is based on the Friis transmission formula. The tissue-equivalent liquid is one of the conducting media so that the effect of the attenuation should be included in the formula. As shown in Fig. 1, two identical antennas are assumed to be immersed and faced in the liquid with perfect polarization. The two antennas are connected to the ports of the vector network analyzer via semi-rigid cables. Looking
The dielectric constant and conductivity of the liquid, $\varepsilon_r$ and $\sigma$, can be determined by using the attenuation and phase constants $\alpha$ and $\beta$ as follows:

$$e_r = \frac{\beta^2 - \alpha^2}{\alpha^2 \mu_0 \varepsilon_0}$$  \hspace{1cm} (6)

$$\sigma = \frac{2\alpha \beta}{\omega \mu_0}$$  \hspace{1cm} (7)

where $\omega$ is the angular frequency, $\mu_0$ and $\varepsilon_0$ are permeability and permittivity in free space.

As the distance, $r$, is longer, the value of $|S_{21}|$ is smaller so that the uncertainty of measured $S_{21}$ is larger. Actually, the uncertainty of measurement should be included in the regression process. If the uncertainty of $|S_{21}(r)|_{dB}$ is denoted as $u_w$, the weight at the distance $r = r_i$ can be selected as $w_i^p = 1/u_i^2$. Then, the figure-of-merit function, or, $\chi$-squares for this regression analysis can be defined as

$$\chi_a^2 = \sum_i w_i^p \left( |S_{21}(r_i)|_{dB, measured} - |S_{21}(r_i)|_{dB} \right)^2$$

$$= \sum_i w_i^p \left( |S_{21}(r_i)|_{dB, measured} + 20 \log_{10} r_i + 8.686r_i - \sum_{m=0}^n A_{m, i} \right)^2$$  \hspace{1cm} (8)

where $|S_{21}(r_i)|_{dB, measured}$ is a measurand of $|S_{21}(r_i)|_{dB}$. If the uncertainty of the variable, $r_i$, can be ignored, $20 \log_{10} r_i$ in (3) cannot generate the uncertainty itself and can be treated as a known function. Therefore, $|S_{21}(r_i)|_{dB, measured} + 20 \log_{10} r_i$ in (8) can be considered as a measurand. Then, the constants in (8), $\alpha, A_m$, can be determined by the method of the linear least-squares. The conditions of $\partial \chi_a^2/\partial \alpha = 0$ and $\partial \chi_a^2/\partial A_m = 0$ lead to the normal equations to determine the constants. The uncertainty of the solutions, $\alpha, A_m$, can be given by the square of the diagonal component in the inverse matrix of the coefficient matrix for the normal equations [8].

Similarly, the uncertainty of measured $\angle S_{21}$ is larger when the level of $|S_{21}|$ is smaller. The method of least squares can be applied if the weight at the distance $r = r_i$ is selected as the square of inverse of the distance. If the uncertainty of $\angle S_{21}(r)$ is denoted as $u_p$ which is generally given as a function of $|S_{21}(r)|_{dB}$, the weight at the distance $r = r_i$ can be selected as $w_i^p = 1/u_i^2$. Then, the figure-of-merit function, or, $\chi$-squares for this regression can be defined as

$$\chi_p^2 = \sum_i w_i^p \left( \angle S_{21}(r_i)_{measured} - \angle S_{21}(r_i) \right)^2$$

$$= \sum_i w_i^p \left( \angle S_{21}(r_i)_{measured} + \beta r_i - \sum_{m=0}^n B_{m, i} \right)^2$$  \hspace{1cm} (9)

where $\angle S_{21}(r_i)_{measured}$ is a measurand of $\angle S_{21}(r_i)$ at the distance $r = r_i$. The conditions $\partial \chi_p^2/\partial \beta = 0$ and $\partial \chi_p^2/\partial B_m$, lead to the normal equations to determine the constants. The uncertainty of the solutions, $\beta, B_m$, can be given by the square

...
of the diagonal component in the inverse matrix of the coefficient for the normal equations [8].

The combined standard uncertainty of the far-field gain, \( u(G_{\text{dB}}) \), should be estimated by using the uncertainty of the constant, \( A_0 \), the phase constant, \( \beta \), and the magnitudes of the reflection coefficients of the antennas, \( |S_{ii}| \), \( i = 1, 2 \), according to (5).

\[
u^2(G_{\text{dB}}) = \left( \frac{\partial G_{\text{dB}}}{\partial A_0} \right)^2 u^2(A_0) + \left( \frac{\partial G_{\text{dB}}}{\partial \beta} \right)^2 u^2(\beta) + \left( \frac{\partial G_{\text{dB}}}{\partial |S_{11}|} \right)^2 u^2(|S_{11}|) + \left( \frac{\partial G_{\text{dB}}}{\partial |S_{22}|} \right)^2 u^2(|S_{22}|) = (0.5u(A_0))^2 + \left( \frac{4.343u(\beta)}{\beta} \right)^2 + \left( \frac{4.343|S_{11}|u(|S_{11}|)}{1 - |S_{11}|^2} \right)^2 + \left( \frac{4.343|S_{22}|u(|S_{22}|)}{1 - |S_{22}|^2} \right)^2. \tag{10}
\]

The fractional standard uncertainty of the gain, \( u(G) / G \), can be related to the uncertainty of the gain in dB representation, \( u(G_{\text{dB}}) \) as follows:

\[
u \frac{G}{G} = 0.2303u(G_{\text{dB}}). \tag{11}
\]

Also, the combined standard uncertainty of the dielectric constant, \( \varepsilon_r \), and the conductivity, \( \sigma \), of the liquid are given by

\[
u \frac{\varepsilon_r}{\varepsilon_r} = \left( \frac{2\alpha}{\beta^2 - \alpha^2} \right)^2 u^2(\alpha) + \left( \frac{2\beta}{\beta^2 - \alpha^2} \right)^2 u^2(\beta), \tag{12}
\]

\[
u \frac{\sigma}{\sigma} = \left( \frac{u(\alpha)}{\alpha} \right)^2 + \frac{u(\beta)}{\beta} \tag{13}
\]

where \( u(\alpha) \) and \( u(\beta) \) can be evaluated through the regression of (8) and (9).

Needless to say, the above discussion is valid for the lower order asymptotic expansion in (3) and (4). Therefore, the following three cases are dealt with in this paper:

(a) \( n = 0: A_0 \neq 0, A_m = 0, m = 1, 2, \cdots \),

\( B_0 \neq 0, B_m = 0, m = 1, 2, \cdots \)

(b) \( n = 1: A_0, A_1 \neq 0, A_m = 0, m = 2, 3, \cdots \),

\( B_0, B_1 \neq 0, B_m = 0, m = 2, 3, \cdots \)

(c) \( n = 2: A_0, A_1, A_2 \neq 0, A_m = 0, m = 3, 4, \cdots \),

\( B_0, B_1, B_2 \neq 0, B_m = 0, m = 3, 4, \cdots \)

The case of (a) corresponds to the procedure to determine the gain and dielectric property of the liquid in the far-field region as discussed before [4]. The estimated far-field gain is sensitive to the choice of the asymptotic terms as well as the fitting range. Especially, the term of \( A_2 / r^2 \) in (3) expresses the contribution of the extremely near-field of the antenna so that it is effective for curve-fitting extremely in the neighborhood of the antenna whereas some ill-conditioned behaviors can be observed in the far-field region. Therefore, we should examine the choice of the asymptotic terms and the fitting range.

3. Experimental Results for Gain and Uncertainty

The conducting medium assumed in this paper is the tissue-equivalent liquid for SAR estimation at 2.45 GHz [2],[9]. Fig. 2 shows our measurement setup. A rectangular tank is filled with about 50l of the tissue-equivalent liquid. The liquid should have a permittivity of 39.2 and conductivity of 1.83S/m at 21°C according to the measurement of the contact probe method. The transmitting antenna is fixed, while the receiving antenna is moved with a stage operated by its controller. To avoid connections through the liquid, the antennas are located at the ends of semi-rigid cables, as shown in Fig. 3. Two offset dipole antennas with a length of \( l = 13\text{mm} \) are used in our measurement. No balun is used because the unbalanced current would be rapidly degraded in the liquid [5]. \( S_{21} \) were measured in the range of \( r = 0 \text{ mm} \) to 200 mm in increment of 1mm, then \( S_{11} \) and \( S_{22} \) were measured at \( r = 200 \text{ mm} \). The values of the return loss of the offset-fed dipole antenna were \(-14.1 \text{ dB} \) and \(-18.8 \text{ dB} \). The output power level of the network analyzer (Agilent N5230A) were

Figure 2 Measurement setup. Two reference antennas in the tissue-equivalent liquid are connected to the slide stage so as to control their separation.

Figure 3 Offset-fed dipole antennas and semi-rigid cables.
set to $-5 \text{ dBm}$.

Figure 4 shows measured $|S_{21}|$ as a function of the distance, $r$. As discussed in [5], the empirical far-field boundary is given by $S(\lambda_e/2) = 5(9.8) = 48 \text{ mm}$ at 2.45 GHz, where $\lambda_e$ is the effective wavelength in the liquid. For $r > 120 \text{ mm}$, the measured $S_{21}$ are fluctuated due to the noise of the measurement system. In general, when the distance between the antennas, $r$, is beyond the far-field boundary, the magnitude of $S_{21}$ is lower than the noise level of the measurement system so that it is difficult to measure $S_{21}$ in the far-field region. This fact suggests that the measurement of $S_{21}$ cannot be implemented in the far-field region, that is, can be implemented in the near-field region. As shown in Fig. 4, the fluctuation is larger as the distance, $r$, is longer. This is the reason why the curve fitting should be required to use the weighted regression.

Table 1 shows some estimated far-field gains and corresponding fractional uncertainties for various regression functions and fitting ranges. For the description of $S_{21}$ in the table, $n$ means that we select $\alpha, A_0, \cdots, A_n$ and $\beta, B_0, \cdots, B_n$ as the parameters of the regression functions in (3) and (4). The uncertainties of $S$ parameters can be evaluated by the worksheet provided by the manufacturer of the instrument. In the table, estimated far-field gain, electric property of the liquid and fractional combined standard uncertainty for the regression analysis based on the Fresnel region gain [7] are also listed. For example, the estimated gain has a value of $-1.72 \text{ dBi}$ for the fitting range of $1 \sim 120 \text{ mm}$. However, the gain estimated by the $S_{21}$ regression greatly depends upon the number of $n$ and the fitting range. As one of the reasons of these scattering results, the regression function should be selected with careful regard to its behavior in the fitting range. For $n = 2$, the fitting range should include the contribution of the extremely near-field and exclude the contribution of the far-field effect because the asymptotic expansion is valid in the limited range. In this case, we should choose $1 \text{ mm}$ as the start of the fitting range, $r_{\text{start}}$, and $n = 2$. Furthermore, the uncertainty is larger as $n$ is larger and $r_{\text{start}}$ is larger. Especially, the choice of $r_{\text{start}}$ includes serious problem affecting the uncertainty as seen from Table 1.

Next, we consider the effect of introducing the weighted least squares. Figs. 5 and 6 show the estimated far-field gain and corresponding fractional uncertainty as a function of the upper limit of the fitting range, $r_{\text{stop}}$ for various $n$ and $r_{\text{start}}$. For comparison, the figures include the results for the regression with/without weight. In the equally-weighted regression, the uncertainty of the measurable is selected as one at the center of the fitting range. As shown in Fig. 5, as $r_{\text{stop}}$ is larger, the estimated gain is more fluctuated and not converge with its true far-field gain for the equally-weighted regression, whereas little variation of the estimated gain can be observed for the weighted regression. Therefore, the weighted regression is effective for our gain estimation in the liquid.

The reason why the curves of the estimated gain for the cases of (b) $n = 1$, $r_{\text{start}} = 30 \text{ mm}$ and (c) $n = 0$, $r_{\text{start}} = 50 \text{ mm}$ do not coincide with the curve for case of (a) $n = 2$, $r_{\text{start}} = 1 \text{ mm}$ is that the regression function for $n = 2$ includes the behavior of the extremely near-field and Fresnel field radiated by the antenna in the liquid. Of course, the regression function for $n = 1$ includes the behavior of the Fresnel field but does not include the behavior of the extremely near-field. In this case, including the extremely near-field is effective for the regression process. Thus, the nature of the regression function should be affected in the fitting range so that it is required to make a deliberate choice of the fitting range as well as the order of the regression.

![Figure 4](image-url) Measured $|S_{21}|_{\text{dBm}}$ and corresponding uncertainty at 2.45 GHz.

<table>
<thead>
<tr>
<th>Regression function</th>
<th>Parameter</th>
<th>1~120 mm</th>
<th>10~120 mm</th>
<th>30~120 mm</th>
<th>50~120 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{21}(r), n = 0$</td>
<td>$</td>
<td>S_{21}</td>
<td>$</td>
<td>$-4.23 \text{ dB} (0.38%)$</td>
<td>$-2.78 \text{ dB} (0.51%)$</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>$36.3 (0.083%)$</td>
<td>$37.1 (0.11%)$</td>
<td>$37.6 (0.20%)$</td>
<td>$37.8 (0.49%)$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\text{cal}}$</td>
<td>$1.39 \text{ S/m} (0.25%)$</td>
<td>$1.67 \text{ S/m} (0.27%)$</td>
<td>$1.77 \text{ S/m} (0.50%)$</td>
<td>$1.80 \text{ S/m} (1.19%)$</td>
</tr>
<tr>
<td>$S_{21}(r), n = 1$</td>
<td>$</td>
<td>S_{21}</td>
<td>$</td>
<td>$-2.57 \text{ dB} (0.49%)$</td>
<td>$-1.62 \text{ dB} (1.62%)$</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>$36.9 (0.10%)$</td>
<td>$37.8 (0.21%)$</td>
<td>$38.0 (0.71%)$</td>
<td>$38.0 (2.75%)$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\text{cal}}$</td>
<td>$1.67 \text{ S/m} (0.25%)$</td>
<td>$1.82 \text{ S/m} (0.51%)$</td>
<td>$1.82 \text{ S/m} (1.68%)$</td>
<td>$1.83 \text{ S/m} (5.75%)$</td>
</tr>
<tr>
<td>$S_{21}(r), n = 2$</td>
<td>$</td>
<td>S_{21}</td>
<td>$</td>
<td>$-1.79 \text{ dB} (0.72%)$</td>
<td>$-1.58 \text{ dB} (4.57%)$</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td>$37.5 (0.12%)$</td>
<td>$38.0 (0.42%)$</td>
<td>$37.9 (2.40%)$</td>
<td>$37.6 (14.0%)$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\text{cal}}$</td>
<td>$1.79 \text{ S/m} (0.30%)$</td>
<td>$1.83 \text{ S/m} (0.99%)$</td>
<td>$1.83 \text{ S/m} (5.17%)$</td>
<td>$1.75 \text{ S/m} (26.6%)$</td>
</tr>
</tbody>
</table>

| Fresnel region gain, $n = 2$ | $|S_{21}|$  | $-1.72 \text{ dB} (1.07\%)$ | $-1.78 \text{ dB} (3.49\%)$ | $-1.92 \text{ dB} (15.3\%)$ | $-1.93 \text{ dB} (53.1\%)$ |
(a) $n = 2$, $r_{\text{start}} = 1 \text{ mm}$.

(b) $n = 1$, $r_{\text{start}} = 30 \text{ mm}$.

(c) $n = 0$, $r_{\text{start}} = 50 \text{ mm}$.

Fig. 5 Variation in estimated far-field gain as a function of upper limit of the fitting range, $r_{\text{stop}}$.

As shown in Fig. 6, the introduction of the weight into the regression analysis hardly influences the uncertainty of the estimated gain for $r_{\text{stop}} > 70 \text{ mm}$ in the case of (a) $n = 2$, $r_{\text{start}} = 1 \text{ mm}$ and for $r_{\text{stop}} > 100 \text{ mm}$ in the cases of (b) $n = 1$, $r_{\text{start}} = 30 \text{ mm}$ and (c) $n = 0$, $r_{\text{start}} = 50 \text{ mm}$. For the weighted regression, the uncertainty does not increase when $r_{\text{stop}}$ is longer, because of preconsidering the weight of the regression. However, the uncertainty of $S_{21}$ is larger so that the uncertainty for the equally-weighted regression tends to be more degraded, as $r_{\text{stop}}$ is longer. Moreover, as the number of terms of the regression function, $n$, is larger, the contribution of the Fresnel field and extremely near-field would lead to increasing the uncertainty. As seen from Table 1 and Fig. 6, we can find that narrowing down the fitting range leads to increasing the uncertainty. Of course, the introduction of the terms of the Fresnel field and extremely near-field is effective for the curve-fitting in the Fresnel field region. However, it leads to increasing the uncertainty. This contradiction is because the behavior of the regression function including the terms of the Fresnel field and extremely near-field has a more complexity than only the far-field terms be-
cause of existing the asymptotic terms.

As shown in Table 1, the estimated dielectric constants of the liquid are a bit smaller than the measured value, 38.9. Also, the estimated conductivities are slightly smaller than the measured value, 1.83 S/m. This is because the attenuation and phase constants can be exactly estimated in the far-field region.

As discussed above, the fluctuation or uncertainty of the estimated gain can be reduced by using the extended Friis transmission formula which is applicable in the extremely near-field region within the liquid and weighted curve-fitting technique. Over 3 GHz, the attenuation in the tissue-equivalent liquid is much larger than that at 2.45 GHz so that the measurable range is narrower as well as shorter effective wavelength in the liquid at higher frequency. As the frequency is higher, the boundary between the far-field region and Fresnel region is shorter so that the estimated gain is more affected by the noise floor of the measurement system and the uncertainty of the system is larger. This means that the uncertainty of $S_{21}$ and the far-field gain does more increase as the distance between the antennas. Therefore, the introduction of the terms of the Fresnel field and extremely near-field and the weighted regression proposed in this paper would be available for calibrating the electric-field probes used in the standardized SAR assessment over 3 GHz.

4. Conclusion

The electric-field probes used in the standard SAR evaluation for the mobile communication devices can be calibrated by the procedure using no waveguide system. For this probe calibration, it is required to calibrate the gain of the reference antenna operated in the tissue-equivalent liquid. The gain as well as dielectric property in the liquid can be simultaneously evaluated by using the extended Friis transmission formula and the method of weighted least squares. If the start of the fitting range is carefully selected, the introduction of the weight in the regression process has the estimated gain little fluctuated for various selections of the stop of the fitting range. In the $S_{21}$ regression, it is also important to select the regression function by considering the behavior of the field radiated by the antenna. And the uncertainty of the gain determined by the weighted regression is estimated.

References

[2] IEC International Standard 62209-1, “Human exposure to radio frequency fields from hand-held and body-mounted wireless communication devices — Human models, instrumentation, and procedures — Part 1: Procedure to determine the specific absorption rate (SAR) for hand-held devices used in close proximity to the ear (frequency range of 300 MHz to 3 GHz),” 2005.
Naoto Ikarashi received B.S. degree in Niigata University in 2007. He is currently a graduate student of Niigata University. He engages in the development of the system for calibrating SAR probes.

Ken-ichi Sato received the B.E. and M.E. degrees from Tamagawa University, Tokyo, Japan in 1993 and 1995, respectively. He joined NTT Advanced Technology Corporation, Tokyo, Japan in 1995. Since 1999, he is working in the field of the electromagnetic compatibility (EMC) problems including biomedical EMC issues, particularly in the Specific Absorption Radio (SAR) measurement.

Lira Hamada received the Ph.D. degree from Chiba University, Chiba, Japan, in 2000. From 2000 to 2005, she was with the Department of Electronics Engineering, University of Electro-Communications, Chofu, Tokyo, Japan. Since 2005, she has been with the National Institute of Information and Communications Technology (NICT), Koganei, Tokyo, Japan. She is a responsible researcher for the measurement and calibration technique for the SAR evaluation system. Dr. Hamada is a member of the IEE of Japan, the IEEE, and the Bioelectromagnetics Society.

Soichi Watanabe received the B.E., M.E., and D.E., degrees in electrical engineering from Tokyo Metropolitan University, Tokyo, Japan, in 1991, 1993, and 1996, respectively. He is currently with the National Institute of Information and Communications Technology (NICT), Tokyo Japan. His main interest is research on biomedical electromagnetic compatibility. Dr. Watanabe is a member of the Institute of Electrical Engineers (IEEE), Japan, the IEEE, and the Bioelectromagnetics Society. He was the recipient of several awards, including the 1996 International Scientific Radio Union (URSI) Young Scientist Award and 1997 Best Paper Award presented by the IEICE. He is also a member of Standing Committee III of International Commission on Non-Ionizing Radiation Protection (IC-NIRP/SCIII) since 2005.