Error Analysis for Ultra-Wideband DS- and Hybrid DS/TH-CDMA with Arbitrary Chip-Duty*

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SUMMARY In this paper, ultra-wideband (UWB) multiple access systems are introduced by using direct-sequence (DS) and hybrid direct-sequence time-hopping (DS/TH) code division multiple access (CDMA) that use arbitrary chip-duty of the spreading sequences. The bit error probabilities are presented. First of all, the variances of the multiple access interference are developed by investigating the collision properties of the signals. Afterward, various approximations are applied. The standard Gaussian approximation (SGA) for the DS system is shown to become extremely optimistic as the chip-duty becomes low. Though the hybrid system performs better, the SGA still remains optimistic. To obtain accurate results, Holtzman’s simplified improved Gaussian approximation (SIGA) and Morrow and Lehnert’s improved Gaussian approximation (IGA) are used. A shortcoming of the SIGA is rediscovered that renders it unusable for low-duty DS systems, especially, at high signal-to-noise ratio. However, for the hybrid system, the SIGA works as an excellent tool. The IGA is used to get accurate results for the low-duty DS systems. It is shown that lowering of chip-duty by keeping chip rate and chip length unchanged improves performance for asynchronous DS and both asynchronous and synchronous hybrid systems. However, under the same processing gain, a high-duty system performs better than a low-duty system. Performance of synchronous DS system remains independent of chip-duty.

**key words:** ultra-wideband, code division multiple access, error analysis, standard and improved Gaussian approximations

1. Introduction

The newly emerging ultra-wideband (UWB) communication is one of the most prospective technologies for future wireless multiple access communications. Since from the inception of UWB [1], the time hopping (TH) impulse radio (IR) UWB systems combined with pulse position modulation (PPM) have been on the focus for last few years. Multiple access analysis of such IR UWB systems has been widely studied (i.e. see [1]–[8] and the references therein). One drawback of TH-IR PPM system is that it transmits unipolar pulses that cause spectral lines in the frequency spectrum [9]. The use of bipolar signaling and bipolar modulation as binary phase shift keying (BPSK) are the right remedy for this problem.

Though many succeeding works on UWB have considered TH-IR PPM systems following the pioneer works of [1], [2], multiple access analysis of UWB systems using bipolar signaling has not received much attention. For instance, the well-known concept of conventional direct-sequence code division multiple access (DS-CDMA) communication can be a key base idea to develop UWB systems with bipolar signaling. One basic difference of conventional DS-CDMA from a UWB system is that the UWB system usually transmits signals of low duty-cycle, whereas, the conventional DS-CDMA transmits signal continuously rendering the signal duty-cycle to be unity. As a consequence, while viewing the conventional DS-CDMA from the perspective of UWB multiple access systems, UWB DS-CDMA systems can be designed by lowering the chip-duty of spreading sequences that is unity in conventional DS-CDMA. While the chip-duty is low, it additionally provides a new option for randomizing the position of the pulse within each chip giving the UWB hybrid DS/TH-CDMA system. Error analysis of such UWB DS- and hybrid DS/TH-CDMA systems are of practical interest, however, haven’t yet been addressed, except in some of our recent works [10]–[12].

As the signal duty goes low, the probability that the signals of simultaneous interfering users will collide with that of the desired user decreases, decreasing the probability of interference [1]. Hence, the multiple access performance of a system is supposed to depend on the collision property of the signal, however though the dependency is not studied yet. Notably, the collision property of signals depend on its structure, which is considerably different for DS and DS/TH. The collision property is of importance both under perfect and imperfect power control and is supposed to bring about a tradeoff in the choice of signaling [1]. However, any study comparing the collision properties and hence the multiple access performances of low-duty DS and hybrid DS/TH sequences is not yet known.

The error analysis for conventional DS-CDMA using random sequences is present in current literature [13]–[15], which is for systems using chip-duty of unity. In [14] Morrow and Lehnert showed the standard Gaussian approximation (SGA) to be optimistic and proposed an accurate improved Gaussian approximation (IGA) with greater computational complexity. Later, Holtzman in [15] proposed a simplified improved Gaussian approximation (SIGA) that provides good accuracy with computational complexity comparable to the SGA. Following the SGA method, error probabilities for some UWB systems were presented in [1], [2], [16]–[18]. However, the SGA for UWB may not be accurate in most cases of practical interest [3]–[7]. How different UWB system parameters including chip-duty affects
the accuracy of SGA needs investigation. In addition, since the UWB signaling structure is considerably different from that of conventional DS-CDMA, whether the simple method SIGA can be applied for UWB is not clear. Otherwise, employing the IGA would be necessary, the use of which for UWB hasn’t yet been investigated either.

In this paper, we present a simple error analysis applicable for both UWB DS- and hybrid DS/TH-CDMA. We consider a UWB DS-CDMA system with arbitrary chip-duty \( \delta \) of the spreading sequences used, \( 0 < \delta \leq 1 \) and a UWB hybrid DS/TH-CDMA system with \( \delta = 1/\iota \), \( \iota \) being an integer \( \geq 2 \). The error analysis is presented from the perspective of collision properties of the sequences. This facilitates to obtain error probabilities following SGA, SIGA and IGA, along with giving us way for investigating the validity of the methods for UWB. Effects of various system parameters including chip-duty on system performance are investigated and the multiple access capabilities of arbitrary-duty DS and DS/TH sequences are compared. To make the analysis tractable, rectangular pulse shape and random sequences are considered. The assumption of rectangular pulse helps us obtain closed form error probabilities. Note that these are standard assumptions in literature [6], [8], [11], [13]–[15], [19], [20]. Monte Carlo simulations are also presented to support the theory.

2. System Model

The binary UWB DS- and UWB hybrid DS/TH-CDMA systems under consideration has \( K \) simultaneous users. The signal of the \( k \)th user can be given by

\[
s_k(t) = \sum_{l=-\infty}^{+\infty} \sqrt{2P_k} a_k(t - lT_b - \alpha_k) \times b_k(t - lT_b - \alpha_k) \cos(\omega_c t + \phi_k)
\]

where \( P_k \) is the average signal power of user \( k \), \( \omega_c \) is the carrier frequency and \( b_k(t) \), \( a_k(t) \) are the data sequence and the spreading chip sequence respectively that are independent of each other. Here \( l \) represents \( l \)-th bit and assume \( k \in [0, 1, \ldots, K - 1] \), \( \alpha_l \) and \( \phi_l \) are random delay and phase respectively relative to the desired signal of the \( l \)th user (i.e., \( \alpha_0 = 0, \phi_0 = 0 \)). \( a_k \) and \( \phi_k \) are independent and uniformly distributed over \([0, T_b] \) and \([0, 2\pi] \) respectively, where \( T_b \) is the bit duration. \( b_k(t) \) is a random process having outcomes uniform on \([+1, -1] \). \( a_k(t) \) over each bit duration is defined as

\[
a_k(t) = \sum_{i=0}^{N-1} a_{k,i} \psi(t - iT_c - C_{i,k}T_p)
\]

where \( \psi(t) \) is the chip waveform, \( T_c \) is the chip duration and \( T_p \) is the duration of signal transmission over each chip. \( C_{i,k} \) is a random variable (RV) controlling the time hopping in the \( i \)th chip of the \( k \)th user in the hybrid DS/TH/CDMA system that equals to zero for the DS system. Assuming a rectangular pulse shape \( \psi(t) = 1, \ 0 \leq t < T_p \) and \( \psi(t) = 0 \), otherwise.

In both the systems, the chip-duty is defined as \( \delta = T_p/T_c \). For the DS system we consider \( 0 \leq \delta < 1 \) and for the hybrid system \( \delta = 1/\iota \), \( \iota \) being an integer \( \geq 2 \). As a result, \( C_{i,k} \) in the hybrid system is considered as a random process uniform on \([0, 1, \ldots, I - 1] \). The energy of \( \psi(t) \) is \( T_c \) and so, \( a_k \) is a random process uniform on \([+\delta^{-1/2}, -\delta^{-1/2}] \). Additionally, \( a_k \) and \( C_{i,k} \) in (2) are periodic with period \( N = T_b/T_c \) and hence the processing gain \( PG = N/\delta \). A diagram of the signaling structure is shown in Fig. 1.

Here note that a term pulse repetition frequency (PRF) is sometime used in UWB literature. The average PRF for both types of the systems introduced above equals \( 1/T_c \).

3. Multiple Access Interference Modeling

Considering an additive white Gaussian noise (AWGN) channel, the received signal can be given by

\[
r(t) = \sum_{k=0}^{K-1} s_k(t - \alpha_k) + n(t)
\]

where \( n(t) \) is AWGN noise with two sided power spectral density of \( N_0/2 \). The time asynchronism of the random sequence of the \( k \)th user can be written as \( \alpha_k = \tau_k + \gamma_k T_c \). Here, \( \gamma_k \) is an integer within \([0, 1, \ldots, N - 1] \) and \( 0 \leq \tau_k < T_c \). Considering a template of the form

\[
s_0^{temp}(t) = \sum_{i=0}^{N-1} a_{0,i} \psi(t - iT_c - C_{0,i}T_p) \cos(\omega_c t)
\]

the decision statistics for detecting an arbitrary bit of user 0 by a coherent correlation receiver in the UWB DS- or hybrid DS/TH-CDMA system can be given by

\[
\lambda_0 = \sqrt{P_0/2b_0T_b} + \sum_{i=0}^{N-1} \sum_{k=1}^{K-1} \sqrt{P_k/2W_{i,k}} \cos \phi_k + n
\]
where the right side of (5) has three parts, of which the first part is the desired signal component, the second part is the multiple access interference (MAI) component and the third part \( n \) is the AWGN component having variance of \( \sigma_n^2 \approx N_0T_s/4 \). Here, \( W_{ik} \) is the MAI component on the \( i \)th chip of user 0 contributed by user \( k \), given by

\[
W_{ik} = L_{i,k} \hat{R}_g(\tau_k) + M_{i,k} R_\phi(\tau_k)
\]

where \( \hat{R}_g(\tau_k) \) and \( R_\phi(\tau_k) \) are continuous-time normalized partial autocorrelation functions of the chip waveform \( \phi(t) \) defined as

\[
\hat{R}_g(\tau_k) = \frac{1}{T_c} \int_0^{T_c} \phi(t)\phi(t-\tau_k)dt
\]

and

\[
\hat{R}_\phi(\tau_k) = \hat{R}_\phi(T_c - \tau_k).
\]

The system is chip synchronous for \( 0 \) where \( S_{11} \) and \( S_{13} \) are independent RVs uniformly distributed over \([0,1]\). For 0.5 < \( \delta \) \leq 1.0, \( W_{ik} \) shown in (6) simplifies to \( L_{i,k} \hat{R}_g(\tau_k) \) for \( 0 \leq \tau_k < T_c \), 0 for \( T_c < T_p \) and \( M_{i,k} R_\phi(\tau_k) \) for \( T_p < \tau_k < T_c \). Here, the MAI component on the \( i \)th chip of user 0 contributed by user \( k \)

\[
Z_{ik} = \frac{P_k}{2} \cos^2 \phi_k E[W_{ik}^2|\alpha]
\]

where \( Z_{ik} \) is the variance of MAI component on the \( i \)th chip of user 0 coming from user \( k \), given by

\[
Z_k = \sum_{i=0}^{N-1} Z_{ik}
\]

and \( Z_k \) is the variance of MAI component on the \( i \)th chip of duration \( T_p \) placed within 0 to \( T_p \) of each chip of duration \( T_c \) for all users, we can write, \( \hat{R}_g(0) = \frac{1}{T_c} \int_0^{T_c} \phi(t)dt = T_p/\delta \) and \( R_\phi(0) = 0 \) for both cases. Here note that the \((i-\gamma_k)\)th chip of user k collides with the \( i \)th chip of user 0, and a collision of chips results in collision of pulses causing MAI. Hence, for both chip and bit synchronous cases, we can write

\[
Z_{i,k} = \frac{P_k}{2} T_c^2 \cos^2 \phi_k
\]

Since there are \( N \) similar collisions over the bit duration, we get

\[
Z_k = \frac{NT_c^2}{2} \cos^2 \phi_k
\]

Note that \( Z_k \) here is independent of chip-duty.

### 3.1 The UWB DS-CDMA System

For ease of presentation, we first start with the synchronous system and then eventually go for the asynchronous system.

**Synchronous system.** The system is chip synchronous while \( \tau_k = 0 \) and bit synchronous while both \( \tau_k = \gamma_k = 0 \). Because, the rectangular pulse of duration \( T_p \) is placed within 0 to \( T_p \) of each chip of duration \( T_c \) for all users, we can write, \( \hat{R}_g(0) = \frac{1}{T_c} \int_0^{T_c} \phi(t)dt = T_p/\delta \) and \( R_\phi(0) = 0 \) for both cases. Here note that the \((i-\gamma_k)\)th chip of user k collides with the \( i \)th chip of user 0, and a collision of chips results in collision of pulses causing MAI. Hence, for both chip and bit synchronous cases, we can write

\[
Z_{i,k} = \frac{P_k}{2} T_c^2 \cos^2 \phi_k
\]

for 0 < \( \delta \) \leq 0.5 where \( S_{11} \) and \( S_{13} \) are independent RVs uniformly distributed over \([0,\delta]\), \([1-\delta,1]\) respectively. For 0.5 < \( \delta \) \leq 1.0, \( W_{ik} \) shown in (6) simplifies to \( L_{i,k} \hat{R}_g(\tau_k) \) for \( 0 \leq \tau_k < T_c \), 0 for \( T_c < T_p \) and \( M_{i,k} R_\phi(\tau_k) \) for \( T_p < \tau_k < T_c \). Here, the MAI component on the \( i \)th chip of user 0 contributed by user \( k \), \( Z_{ik} \) shown in (6) simplifies to \( L_{i,k} \hat{R}_g(\tau_k) \) for \( 0 \leq \tau_k < T_p \), 0 for \( T_p < \tau_k < T_c \) and remains as it is for \( T_c < T_p < \tau_k < T_c \). Hence,

\[
Z_{ik} = \frac{P_k}{2} T_c^2 \cos^2 \phi_k (\delta - S_{11})^2,
\]

\( 0 \leq \tau_k < T_p \)

\[
= 0,
\]

\( T_p < \tau_k < T_c - T_p \)

\[
= \frac{P_k}{2} T_c^2 \cos^2 \phi_k (S_{13} - 1 + \delta)^2,
\]

\( T_c - T_p \leq \tau_k < T_c \)

for 0 < \( \delta \) \leq 0.5 where \( S_{21}, S_{22}, S_{23} \) and \( S_{23} \) are independent RVs, uniformly distributed over \([0,1-\delta],[1-\delta,\delta]\) and \([\delta,1]\) respectively.

Since the pulse in each chip of each user is placed in the same location for the DS sequences, as is synchronous case, here also we note that a collision means a collision in all \( N \) chips of the bit. Hence,

\[
Z_k = NZ_{i,k}
\]

for 0 < \( \delta \) \leq 1.0.

### 3.2 The UWB Hybrid DS/TH-CDMA System

For the hybrid DS/TH-CDMA system, we notice more interesting collision property of the hybrid DS/TH sequences.

For ease of presentation, we again start with the synchronous system.
Synchronous system. Due to the random time hopping patterns, the pulse in any chip of any user may be placed in any of the I slots. Here note that, the \((i - \gamma_k)\)th chip of user \(k\) coincides with the \(i\)th chip of user 0. However, MAI may occur only if the pulses in the respective chips coincide (or, collide). Hence, by noting that \(R_\gamma(\tau = 0) = T_p/\delta\) and \(R_\gamma(\tau = 0) = 0, \tau = (C_i - \gamma_k - C_i0)T_p\), we get

\[
Z_{i,k} = \begin{cases} \frac{P_k}{2} T_c^2 \cos^2 \phi_k, & C_{i,0} = C_{i - \gamma_k} \\ 0, & \text{otherwise} \end{cases}
\]

(15)

Note that a coincidence in any chip occurs with a probability \(\delta\), the probability of no coincidence being \(1 - \delta\). Because the position of pulse in any chip of any user is selected randomly and independently, we can write

\[
Z_k = \frac{P_k}{2} T_c^2 \cos^2 \phi_k J_s
\]

(16)

where \(J_s\) is an RV binomially distributed over \([0, 1, 2, \ldots, N]\) \[21\]. Here note that \(J_s\) represents the number of chips within a bit of user 0 where the individual pulses from the colliding chips of user \(k\) coincide with the pulses of user 0 and hence contribute to MAI. The probability density function of \(J_s\) is given by \[21\]

\[
f_J(j) = \sum_{j=0}^N \binom{N}{j} p^j (1 - p)^{N-j} \delta_D
\]

(17)

with \(p = \delta\) where \(\delta_D\) is Dirac delta function.

Asynchronous system. For a hybrid asynchronous system, note that the interference on the \(i\)th chip of user 0 may come partially from either \((i - \gamma_k)\)th or \((i - \gamma_k - 1)\)th or from both the chips of any user \(k\). Let us define, \(t_i - \gamma_k = (C_i - \gamma_k)T_p + \tau_k\) and \(t_i - \gamma_k - 1 = (C_i - \gamma_k - 1)T_p + \tau_k - T_c\) - \(C_i0T_p\). Any one of the following four possible cases may occur. Considering that only one pulse either from the \((i - \gamma_k)\)th or \((i - \gamma_k - 1)\)th chip of user \(k\) contributes to interference, we get case a: if \(0 \leq \tau < T_p\), where \(\tau = t_i - \gamma_k\) or \(t_i - \gamma_k - 1\), the partial correlations \(R_\gamma(\tau) = (T_p - \tau)/\delta\) and \(R_\gamma(\tau) = 0\); and case b: if \(-T_p \leq \tau < 0\), where \(\tau = t_i - \gamma_k\) or \(t_i - \gamma_k - 1\), one gets \(R_\gamma(\tau) = (T_p + \tau)/\delta\) and \(R_\gamma(\tau) = 0\). However, when the pulses in \((i - \gamma_k)\)th and \((i - \gamma_k - 1)\)th chips are adjacent and both of them contribute to interference, which may occur case c: while \(0 \leq t_i - \gamma_k < T_p\) giving \(R_\gamma(\tau) = (T_p - \tau)/\delta, R_\gamma(\tau) = \tau/\delta\), where \(\tau = t_i - \gamma_k\). The final possibility is that case d: there may be no collision at all. Accumulating the concept developed above, \(Z_{i,k}\) can be given by

\[
Z_{i,k} = \begin{cases} \left(\frac{P_k}{2}\right) T_c^2 \cos^2 \phi_k (\delta - S_{11}), & 0 \leq \tau < T_p \\ \left(\frac{P_k}{2}\right) T_c^2 \cos^2 \phi_k (\delta + S_{12}), & -T_p \leq \tau < 0 \\ \left(\frac{P_k}{2}\right) T_c^2 \cos^2 \phi_k [\delta^2 - 2\delta S_{11} + 2S_{11}^2], & 0 \leq t_i - \gamma_k < T_p \\ 0, & \text{otherwise} \end{cases}
\]

(18)

where \(\tau = t_i - \gamma_k\) or \(t_i - \gamma_k - 1\) and \(S_{12}\) is an RV, uniformly distributed over \([-\delta, 0]\). Note that the average probability of occurrence of the events \(c, d\) are \(\delta^2, 1 - 2\delta\) respectively and that of \(a, b\) are \((2 - \delta)\delta/2\) each.

Here note that \(Z_{i,k}\) in \(18\) gives the exact statistical representation of the possible MAI on the \(i\)th chip of user 0 from user \(k\). However, use of \(18\) in the succeeding sections seems cumbersome. Hence, we look for an easy but still accurate approximation of \(Z_{i,k}\). Due to the TH nature of the position of pulse within any chip, we note an interesting property that if there is a partial coincidence (of pulse) of amount \(\tau, 0 \leq \tau < T_p\) in any chip, the partial coincidences in other chips of the bit will be either \(\tau\) or \(T_p - \tau\) or there will be no coincidences at all. In addition, there is one pulse in average to interfere with an arbitrary pulse of the desired user. So, \(Z_{i,k}\) can be approximately given by

\[
Z_{i,k} = \left\{ \begin{array}{ll} \frac{P_k}{4\delta^2} T_c^2 \cos^2 \phi_k \left[ \delta^2 - 2\delta S_{11} + 2S_{11}^2 \right] J_{as}, & \text{partial coincidence} \\ 0, & \text{otherwise} \end{array} \right.
\]

(19)

Note that a coincidence in any chip occurs with a probability \(2\delta\), the probability of no coincidence being \(1 - 2\delta\). Using this information and those developed in the synchronous case above, \(Z_k\) for \(I \geq 3\) can be approximately given by

\[
Z_k = \frac{P_k}{4\delta^2} T_c^2 \cos^2 \phi_k \left[ \delta^2 - 2\delta S_{11} + 2S_{11}^2 \right] J_{as}
\]

(20)

where \(J_{as}\) is an RV binomially distributed over \([0, 1, \ldots, N]\). As before, \(J_{as}\) represents the number of partial collisions within a bit of user 0 where the pulses of user 0 are struck partially by pulses from the signal of user \(k\). The probability density function of \(J_{as}\) is given by \(17\) with \(p = 2\delta\). For \(I = 2\), using \(18\) (in \(8\)) is recommended.

4. Error Probabilities

With the representations of the MAI variance derived in the previous section, we are now ready to develop the bit error probability (BEP) equations for the systems under consideration. Because of its simplicity and wide acceptance in literature, let us start with the standard Gaussian approximation (SGA).

4.1 Standard Gaussian Approximation (SGA)

The SGA is simple in the sense that once we get the mean of the MAI variance, we can obtain the BEP.

4.1.1 The UWB DS-CDMA System

Synchronous system. For synchronous system, the mean of the MAI variance \(\Psi\) can be given by,

\[
\mu = \sum_{k=1}^{K-1} E[Z_k] = \frac{N T_p^2}{4} \sum_{k=1}^{K-1} P_k
\]

(21)
for $0 < \delta \leq 1$. The BEP under SGA is then given by

$$P^\text{SGA}_e = Q \left( \frac{P_0 T_h^2 / 2}{\mu + \sigma_n^2} \right)$$

$$= Q \left( \left\{ \frac{\chi N P_0}{\sum_{k=1}^{K-1} P_k} \right\}^{-1} + \left( \frac{2 E_b}{N_0} \right)^{-1} \right)^\frac{1}{2}$$

(22)

where $Q(x) = (2\pi)^{-1/2} \int_x^\infty \exp(-u^2/2)du$, energy per bit of the desired user $E_b = P_0 T_h$, and $\chi = 2$, $0 < \delta \leq 1$.

Asynchronous system. According to Appendix A, by noting that

$$E[S_{11}] = \delta^2 / 2,$$
$$E[S_{11}^2] = \delta^2 / 3,$$
$$E[S_{13}] = 1 - \delta^2 / 2,$$
$$E[S_{21}] = 1 - \delta + \delta^2 / 3,$$
$$E[S_{22}] = (1 - \delta) / 2,$$
$$E[S_{23}] = (1 + \delta + \delta^2) / 3,$$

the mean of $\Psi$ can be given by,

$$\mu = \frac{N T_h^2 \delta}{6} \sum_{k=1}^{K-1} P_k$$

(23)

for $0 < \delta \leq 1$. Finally, the BEP is given by (22) with $\chi = 3/\delta$, $0 < \delta \leq 1$.

4.1.2 The UWB Hybrid DS/TH-CDMA System

Synchronous system. For a hybrid synchronous system, by noting that $E[J_n] = N\delta$ [21], the mean of $\Psi$ is given by

$$\mu = \frac{N T_h^2 \delta}{4} \sum_{k=1}^{K-1} P_k$$

(24)

The BEP can be given by (22) with $\chi = 2/\delta$, $\delta = 1/I$, $I = 2, 3 \ldots$

Asynchronous system. In a similar fashion, using the fact $E[J_n] = 2N\delta$ [21] in (20), or (18) in (8), the mean in this case takes the form as shown in (23). The BEP is given by (22) with $\chi = \delta / 3$, $\delta = 1/I$, $I = 2, 3 \ldots$ Here note that in asynchronous case, both DS and DS/TH systems have the same expression for the mean of MAI variance, which results in the same BEP expression for the systems under SGA.

4.1.3 Remarks

Because of its simplicity, the SGA has been widely used in literature to obtain BEP for conventional DS-CDMA in early works [13]–[15], [19], [20]. Many recent works on UWB multiple access systems have also been performed under the SGA [1], [2], [16], [17], however, without due justification. Moreover, the effects of UWB system parameters, especially, chip-duty and power levels of the users including the effect of the signaling scheme (DS or DS/TH) on the accuracy of the SGA has not yet been investigated. Though the validity of the SGA for UWB has not yet been fully explored, some works based on exact analysis [4] that are usually unwieldy and simulation [7] that are not reusable, show the inefficiency of the SGA. At this situation, a simple to evaluate and accurate method of error analysis is demanding for UWB multiple access system. Hence, in the next stage we proceed for an accurate method for BEP evaluation without surrendering the simplicity.

4.2 Simplified Improved Gaussian Approximation (SIGA)

An acceptably accurate, simple-to-evaluate approximation, known as simplified improved Gaussian approximation (SIGA) was first presented in [15]. This requires the standard deviation $\sigma$ other than the mean $\mu$ of $\Psi$ to be determined. The BEP then becomes

$$P^\text{SIGA}_e = \frac{2}{3} Q \left( \frac{P_0 T_h^2 / 2}{\sqrt{\mu + \sigma_n^2}} \right)$$

$$+ \frac{1}{6} Q \left( \frac{P_0 T_h^2 / 2}{\sqrt{\mu + \sqrt{3} \sigma + \sigma_n^2}} \right)$$

$$+ \frac{1}{6} Q \left( \frac{P_0 T_h^2 / 2}{\sqrt{\mu - \sqrt{3} \sigma + \sigma_n^2}} \right)$$

(25)

4.2.1 The UWB DS-CDMA System

Synchronous system. By neglecting the covariance term of $Z_k$ (that we do all throughout this section) [20], one can show that for a synchronous DS system

$$\sigma = \left[ \sum_{k=1}^{K-1} \left( E[Z_k^2] - E[Z_k^2] \right) \right]^{1/2}$$

$$= \frac{N T_h^2}{4 \sqrt{2}} \left( \sum_{k=1}^{K-1} P_k \right)^{1/2}$$

(26)

for $0 < \delta \leq 1$ which is also independent of $\delta$ like the mean.

Asynchronous system. It can be shown that (see Appendix A),

$$E[S_{11}^3] = \delta^3 / 4,$$
$$E[S_{11}^4] = \delta^4 / 5,$$
$$E[S_{13}^3] = 1 - 3\delta^2 / 2 + \delta^2 - 3\delta^3 / 4,$$
$$E[S_{13}^4] = 1 - 2\delta + 2\delta^2 - \delta^3 + 3\delta^4 / 5,$$
$$E[S_{21}^3] = (1 - \delta)^3 / 4,$$
$$E[S_{21}^4] = (1 - \delta)^4 / 5,$$
Synchronous system. Using (18) and (8) for $N$ terms, especially for small $N$, systems, especially for small $N$, and at moderate to high SNR.

The IGA first introduced in [14] for DS-CDMA is based on the observation that the MAI is approximately Gaussian conditioned on the delays and phases of all the interfering signals. IGA requires the distribution of $\Psi$ to be known, and the BEP is then given by

$$p_{e}^{IGA} = \int_{0}^{\infty} Q_{0} \left( \frac{P_{0}^2 \tau^2}{2} \right) d\zeta$$

where $f_{\Psi}(\zeta)$ is the distribution of $\Psi$.

Finding $f_{\Psi}(\zeta)$ requires that the distribution of $Z_k$ be
determined for all interfering users and then convolved together [14], [20]. It is done with the underlying assumption that the interference from different users are independent.

4.3.1 The UWB DS-CDMA System

Synchronous system. For a synchronous UWB DS system with any chip-duty $0 < \delta \leq 1$, finding the distribution of $Z_k$ is easy (see Appendix B), which is given by

$$f_{Z_k}(z) = \frac{1}{\pi \sqrt{\sigma(Z_k - z)}}, \quad 0 < z < N_k$$

(33)

where $N_k = NP_kT^2/2$. Note that Eq. (33) is independent of $\delta$.

Asynchronous system. Since the collision and interference properties of UWB DS sequence are different for $0 < \delta \leq 0.5$ and $0.5 < \delta \leq 1.0$ in asynchronous mode, the density of MAI variance takes different forms in the two regions. The density of $Z_k$ for $0 < \delta \leq 0.5$ can be shown to be (see Appendix B)

$$f_{Z_k}(z) = \begin{cases} \frac{1}{\pi \sqrt{\sigma(N_k - z)}}, & z = 0 \\ \frac{1}{\pi \sqrt{\sigma(N_k - z)}}, & 0 < z < N_k \end{cases}$$

(34)

where $\delta_D$ is Dirac delta function and $\beta = 1 - 2\delta$. For $0.5 < \delta \leq 1$, we get (see Appendix B)

$$f_{Z_k}(z) = \begin{cases} \frac{1}{\pi \sqrt{\sigma(N_k - z)}}, & 0 < z\frac{1}{\pi \sqrt{\sigma(N_k - z)}} - \frac{1}{\pi \sqrt{\sigma(N_k - z)}} - \frac{1}{\pi \sqrt{\sigma(N_k - z)}} \right), & 0 < z < \frac{N_k(1+D)}{\delta^2} \\ \frac{1}{\pi \sqrt{\sigma(N_k - z)}}, & \frac{N_k(1+D)}{\delta^2} < z \leq N_k \end{cases}$$

(35)

where $D = 4(\delta^2 - \delta)$ and $\beta = 2\delta - 1$.

Here, it is notable that for asynchronous low-duty DS system, the density of MAI variance shown in (34) becomes highly impulsive, which is far away from Gaussian density.

4.3.2 The UWB Hybrid DS/TH-CDMA System

Synchronous system. As we have described in appendix B, the density of $Z_k$ in a UWB hybrid DS/TH synchronous system can be given by

$$f_{Z_k}(z) = \begin{cases} (1 - \delta)^N \delta_D(z), & z = 0 \\ \delta^j \left(1 - \frac{1}{\beta^2} \right)^{N+j} \frac{1}{\pi \sqrt{\sigma(N_k - z)}}, & 0 < z < \frac{jN_k}{N} \end{cases}$$

(36)

Asynchronous system. For the hybrid asynchronous system, finding the actual density of $Z_k$ using (8) and (18) is a tough task. Finding the density of $Z_k$ first and then convolving that $N-1$ times with itself to get the density of $Z_k$, a method that was used in [5], doesn’t provide accurate results in our case. The reason is that the interferences occurring at different chips of the desired user are not independent; rather interferences at adjacent chips are dependent. Accurate density of $Z_k$ cannot be obtained even from (20), because (20) is an expression that can only approximate the mean and the variance of the distribution accurately, not the distribution as a whole. The SGA works excellent and we don’t need the density.

5. Numerical Examples

In this section, the results derived in the previous sections are illustrated by providing specific examples. We present results for equal and unequal power levels of the users. Here recall that the SNR, $E_b/N_0$, mentioned in the next subsections represents the SNR of the desired user 0 with $E_b = P_0T_b$.

5.1 Equal Powers

5.1.1 Performance

With the assumption of perfect power control, in this subsection, we set $P_k = P$ for all users, $k = 0, 1, \ldots, K - 1$. Because the asynchronous systems are more practical, we first present results for such systems. Figure 3 shows BEP vs. SNR, $E_b/N_0$, for asynchronous UWB DS- and hybrid DS/TH-CDMA systems with $N = 20$, chip-duty $\delta = 0.5, 0.25$ and total users $K = 8$. As seen, lowering of chip-duty provides better performance, since $PG = N/\delta$ is obtained. The results are presented from SGA, SIGA and Monte Carlo simulations. The results from the SGA tend to tell us that with similar system parameters such as, chip length $N$, chip-duty $\delta$ etc, both the DS and DS/TH systems

![BEP vs. SNR, $E_b/N_0$ for asynchronous UWB DS- and hybrid DS/TH-CDMA for $N = 20$ and chip-duty $\delta = 0.5, 0.25$ with $K = 8$.]
should show similar performance. However, though this is true at low SNR, the two systems are actually found to perform differently at high SNR. The results from the SIGA match well with those from simulation and can be considered as reliable approximations. Here note that at high SNR, the hybrid system shows better performance than the DS counterpart.

Figure 4 shows BEP vs. total users $K$ for the asynchronous UWB DS- and hybrid DS/TH-CDMA systems with $N = 7$ and chip-duty $\delta = 1.0$ (DS only), 0.5 and 0.1 as shown. The SGA and SIGA are used for both types of the systems. To focus the applicability of SIGA at high SNR, the AWGN is neglected. As seen, the SIGA though excellent for the hybrid system, can only be evaluated beyond a number of users in DS (Table 1) that limits its use for low-duty DS signals. The IGA is used in such cases. The results obtained from the IGA and SIGA are very accurate and match well with those from Monte Carlo simulations. As before, performances of both the systems improve with lowering of chip-duty, because PG is obtained. However, the hybrid system performs better than the DS system for small to medium number of users, and the performance difference between them increases with lowering of chip-duty. In other words, the hybrid system performs closer to the SGA. Because of practical importance, let us now come back to asynchronous system again. Recall that the processing gain of the system is defined as $PG = N/\delta$. Hence, systems may have same PG with different combinations of $N$ and $\delta$. How $N$ and $\delta$ affect the multiple access performance is investigated next. In Fig. 6, we compare the performance of low-duty and high-duty UWB systems under the same PG = 70, with $(N, \delta) = (7, 0.1), (35, 0.5)$ and $E_b/N_0 = \infty$. For small to medium number of users, a high-chip-duty system performs better than a similar low-duty system. However, the performance difference is larger in the case of DS system. Figure 7 shows the BEP vs. $K$ for the synchronous UWB systems. Performance of the DS system remains independent of chip-duty even though PG is obtained at low duty. The reason can be understood intuitively. Since, lowering of chip-duty brings no change in the collision property of synchronous DS spreading sequences, no change in the multiple access performance is seen. So, providing PG by lowering chip-duty is not the right way for a synchronous DS system.
the actual system performance becomes different for different combinations of $N$ and $\delta$, despite having the same $PG$. The $PG$ is increased by 1) decreasing $\delta$ keeping $N$ fixed at 20 and 2) increasing $N$ keeping $\delta$ fixed at 0.5. As seen, better performance is obtained in the second case. This is because, between $N$ and $\delta$, $N$ effects performance more dominantly than $\delta$. Also, notably, the SGA which becomes increasingly optimistic as compared to the SIGA, can neither differentiate between the DS and DS/TH systems, nor can it model the effect of $N$ and $\delta$ under the same $PG$. As seen from SGA, the use of long repetition code brings more performance improvement than decreasing signal duty-cycle ($= \delta$) to increase the $PG$.

In the results presented so far, understanding the role of the PRF of the systems could be of interest. It can be shown that for both the systems, the average PRF $= 1/T_c = N/T_{bp}$. Hence, a fixed data rate implies $N \propto$ PRF. So, the effect of PRF on multiple access performance is proportional to that of $N$.

5.1.2 Remarks on Collision Properties

As we have indicated in the introduction, modeled in the subsequent sections and just evaluate above, the multiple access performance with low-duty signals largely depends on the collision properties of the signals. To investigate the matter further, let us define a term $probability of collision$ (PoC), which represents the probability that the signal of any interfering user collides in some part with that of the desired user. We consider asynchronous systems. The PoC for DS signaling equals $2\delta$. However, note that a collision in this case means a collision in all $N$ pulses of the bit. Unlike this, the probability that there may be collision in any chip equals $2\delta$ for the DS/TH signal. Because the collisions in different chips are independent, the probability that there is no collision at all over the bit duration is $(1 - 2\delta)^N$, giving us the PoC equal to $1 - (1 - 2\delta)^N$. It can be seen that for $\delta < 0.5$, $PoC_{DS/TH} \gg PoC_{DS}$. This more random collisions makes the central limit theorem (CLT) [22] more applicable for DS/TH system. Hence, its performance under perfect power control approaches to the lower bound given by the SGA, resulting in much better performance than that of DS for small to medium number of users. Note that the phenomena described above becomes possible because in binary system, a collision may randomly either aid or degrade the desired user’s signal.

5.2 Unequal Powers

Figures 8 and 9 show the effects of unequal power levels of the users on error performance of the two types of asynchronous systems and on the validity of the SGA. We consider a case where the interfering users (except the desired user 0) use the same power $P_k = P$, $k = 1, 2, \ldots, K - 1$. Figure 8 presents the BEP versus chip-duty $\delta$ for the systems with equal ($P/P_0 = 0$ dB) and unequal ($P/P_0 = 6$ dB) powers of the users. Because for the DS/TH system $\delta = 1/I$, where $I$ can take only integer values $\geq 2$, we present only discrete values in Fig. 8 as we did in Fig. 7. We set $N = 15$
and SNR, $E_b/N_o = 13$ dB and $K = 10$. As seen, while the interfering users have higher power level, system performance is degraded. However, still decreasing chip-duty brings performance improvement in both cases of equal and unequal powers. Here note that while we have presented most of the accurate results of Fig. 8 using SIGA, we were unable to use it for the DS system with $P/P_0 = 6$ dB (see Eq. (31)). The IGA, though a bit more involved, was useful in this case.

From Fig. 8, it is also notable that the performance difference between DS and DS/TH systems, especially at low duty, decreases considerably at unequal power case. To investigate the effect of unequal power levels on the two types of systems further, we present BEP vs. $P/P_0$ in Fig. 9. We set $K = 6$ and SNR, $E_b/N_o = 20$ dB and vary the ratio $P/P_0$ from 0 dB to 10 dB. Here note that the MAI increases with increasing $P/P_0$, and $P/P_0 = 0$ dB stands for the special case of equal powers for all users. Figure 9 shows results both from SIGA and SGA for both the systems with $N = 20$ and $\delta = 0.5$. Though for small values of $P/P_0$, the SGA is optimistic, it’s interesting to see that the SGA is reasonably accurate for $P/P_0 > 6$ dB. Figure 9 also shows that though the DS/TH system performs better for small $P/P_0$, performance of such system is degraded faster as $P/P_0$ increases, until the SIGA touches SGA where both DS and DS/TH systems perform comparably. As a reason for this, the collision properties of the signals are again responsible. As explained in the preceding subsection, the ‘more collision’ and PoC_{DS/TH} $\gg$ PoC_{DS} was the underlying reason for better performance of DS/TH in case of equal powers of all users. Unfortunately, the same ‘more collision’ and PoC_{DS/TH} $\gg$ PoC_{DS} in DS/TH make it more vulnerable to powers coming from other users, a factor that becomes critical in the absence of power control, especially while the interfering users have power levels higher than the desired user.

6. Conclusions

Bit error probabilities have been presented for UWB DS- and hybrid DS/TH-CDMA systems that use spreading sequences with arbitrary chip-duty. The multiple access performances of the systems were compared and the effect of chip-duty on the validity of various approximations used for deriving closed form expressions was investigated. It was shown that under the same processing gain, a high-duty system performs better than a low-duty system. It can be concluded from this paper that if perfect power control can be maintained, a hybrid DS/TH should be the better choice over DS, especially for low-duty signaling. However, if the perfect power control cannot be maintained, use of the hybrid system over DS may not be justified, especially, while the interfering users use higher power levels. A situation with no multipath is considered in this paper. Future research includes analysis over multipath channels.

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References


Appendix A

Finding the means of $S_{11}^n$ and $S_{23}^n$, $n$ being an integer:

Since $S_{11}$ is an RV uniformly distributed over $[0, \delta)$,

$$E[S_{11}^n] = \delta^n E[x_1^n] = \frac{\delta^n}{n+1} \quad (A\cdot1)$$

where $x_1$ is an RV uniformly distributed over $[0, 1]$. In a similar fashion, since $S_{23}$ is an RV uniformly distributed over $[0, 1-\delta]$,  

$$E[S_{23}^n] = (1-\delta)^n E[x_2^n] = \frac{(1-\delta)^n}{n+1} \quad (A\cdot2)$$

By knowing that $S_{11}$, $S_{22}$ and $S_{23}$ are RVs uniformly distributed over $[1-\delta, 1]$, $[1-\delta, \delta]$ and $[\delta, 1]$ respectively, one can write

$$E[S_{11}^n] = E[(1-\delta + S_{11})^n] \quad (A\cdot3)$$

$$E[S_{22}^n] = E[(1-\delta + x_2^n)^n] \quad (A\cdot4)$$

where $x_2$ is an RV uniformly distributed over $[0, 2\delta - 1]$ and

$$E[S_{23}^n] = E[(\delta + S_{23})^n] \quad (A\cdot5)$$

those give the results shown in Sects. 4.1.1 and 4.2.1.

Appendix B

Finding the densities of $Z_k$:

Let us define $N_k = NP_k T_p^2/2$.

Synchronous UWB DS system. (11) gives $Z_k = N_k \cos^2 \phi$ and (33) is easily obtained [21].

Asynchronous UWB DS system. Let us consider that the RV, $Z_k = UV$ where $U$ and $V$ are two independent RVs. The density of $Z_k$ is then given by [14], [21]

$$f_{Z_k}(z) = \int_{v_1}^{v_2} \frac{1}{|v|} f_U(v) f_U(z/v) \, dv \quad (A\cdot6)$$

with $v_1 = -\infty$ and $v_2 = +\infty$ where $f_U(u)$ and $f_U(v)$ are the densities of $U$ and $V$ respectively. Using (12) in (14), $Z_k$ for $0 < \delta \leq 0.5$ is defined by defining, $U = 2 \cos^2 \phi$ and $V = N_k(1-S_{11}/\delta)^2/2$ for $0 \leq \tau_k < T_p$, $V = 0$ for $T_p \leq \tau_k < T_c-T_p$ and $V = N_k(1-1/\delta)^2/2$ for $T_c-T_p \leq \tau_k < T_c$. Following [21], it can be shown that the density of $U$ is given by, $f_U(u) = \pi^{-1}[u(2-u)]^{-1/2}$; $0 < u < 2$ and that of $V$ is given by $f_V(v) = [N_k/2]^{-1/2}$; $0 < v < N_k/2$ for $0 \leq \tau_k < T_p$ and $T_c-T_p \leq \tau_k < T_c$, and $f_V(v) = \delta_{p\delta}$; $v = 0$ for $T_p \leq \tau_k < T_c-T_p$ where $\delta_{p\delta}$ is Dirac delta (impulse) function of unit area. Finally, using (A-6) with $v_1 = 0$, $v_2 = N_k/2$ we get (34).

For $0 < \delta \leq 1$, using (13) in (14) shows that $U$ remains as above and $V$ becomes $V = N_k(1-S_{23}/\delta)^2/2$ for $0 \leq \tau_k < T_c-T_p$, $V = N_k(S_{23}^2 - S_{22}) + (\delta^2 - \delta + 1/2)/(\delta^2)$ for $T_c-T_p \leq \tau_k < T_c$, and $V = N_k(1-1/\delta + S_{23}/\delta)^2/2$ for $T_c-T_p \leq \tau_k < T_c$. Again following [21], the densities can be given by $f_V(v) = [N_kv/2]^{-1/2}$; $N_k(D + 1)/(2\delta^2) < v < N_k/2$ with $D = 4(\delta^2 - \delta)$ for $0 \leq \tau_k < T_c-T_p$ and $T_p \leq \tau_k < T_c$, and $f_V(v) = 2\delta^2[4N_k\delta^2v - N_k^2(D + 1)]^{-1/2}$; $N_k(D + 1)/(4\delta^2) < v < N_k(D + 1)/(2\delta^2)$ for $T_c-T_p \leq \tau_k < T_p$. Finally, following [14] we get (35).

Synchronous UWB DS/TH system. (16) gives $Z_k = UV = (N_k/N) \cos^2 \phi$, $U$ Density of $V = Js_k$ is given by (17) with $p = \delta$, and of $U = (N_k/N) \cos^2 \phi$ by $f_U(u) = \pi^{-1}[u(N_k/N - u)]^{-1/2}$; $0 < u < N_k/N$ [21]. Then using (A-6) we get (36).

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